Homework 2 Solutions

Section 1.3: 3, 8, 15, 18, 24, 41, 44

3) $$\lim_{x\to 0}\left[(2x+1)(x-3)\right] \overset{6}{=} \lim_{x\to 0} 2x+1 \cdot \lim_{x\to 0} x-3$$

$$\overset{4}{=} \left(\lim_{x\to 0} 2x + \lim_{x\to 0} 1\right)\left(\lim_{x\to 0} x - \lim_{x\to 0} 3\right)$$

$$\overset{3}{=} \left(2\lim_{x\to 0} x + \lim_{x\to 0} 1\right)\left(\lim_{x\to 0} x - \lim_{x\to 0} 3\right)$$

$$\overset{2}{=} \left(2\cdot 0 + \lim_{x\to 0} 1\right)\left(0 - \lim_{x\to 0} 3\right)$$

$$\overset{1}{=} (0+1)(0-3) = \boxed{-3}$$

8) $$\lim_{x\to -3}\sqrt{5x^2+2x} \overset{9}{=} \sqrt{\lim_{x\to -3} 5x^2+2x}$$

$$\overset{4}{=} \left[\lim_{x\to -3} 5x^2 + \lim_{x\to -3} 2x\right]^{1/2} \overset{3}{=} \left[5\lim_{x\to -3} x^2 + 2\lim_{x\to -3} x\right]^{1/2}$$

$$\overset{8}{=} \left[5\left(\lim_{x\to -3} x\right)^2 + 2\lim_{x\to -3} x\right]^{1/2} \overset{2}{=} \left[5(-3)^2 + 2(-3)\right]^{1/2}$$

$$= \left[45-6\right]^{1/2} = \boxed{\sqrt{39}}$$

15) $$\lim_{x \to -1} \frac{(x^2 - 2x - 3)}{x+1} = \lim_{x \to -1} \frac{(x-3)(x+1)}{(x+1)} = \lim_{x \to -1} x - 3$$

$$= -1 - 3 = \boxed{-4}$$

18) $$\lim_{x \to 2} \frac{x^2 + 7x + 10}{x+2} = \lim_{x \to 2} \frac{(x+5)(x+2)}{(x+2)} = \lim_{x \to 2} x + 5 = 2 + 5 = \boxed{7}$$

24) $$\lim_{w \to -2} \frac{(w+2)(w^2 - w - 6)}{w^2 + 4w + 4} = \lim_{w \to -2} \frac{(w+2)(w-3)(w+2)}{(w+2)(w+2)}$$

$$= \lim_{w \to -2} w - 3 = -2 - 3 = \boxed{-5}$$

41) $$\lim_{x \to -3^+} \frac{\sqrt{3+x}}{x}$$

note: as we are taking $\lim_{x \to 3^+}$ (the limit as x approaches 3 from the right) the square root is defined

$$\Rightarrow \lim_{x \to -3^+} \frac{\sqrt{3+x}}{x} = \frac{\sqrt{3+(-3)}}{-3} = \frac{\sqrt{0}}{-3} = \boxed{0}$$

44) $$\lim_{x \to 1^-} \frac{\sqrt{1+x}}{4+4x}$$

as $x \to 1^-$, $1 + x \to 2 > 0$ so $\sqrt{1+x}$ is defined

$$\Rightarrow \lim_{x \to 1^-} \frac{\sqrt{1+x}}{4+4x} = \frac{\sqrt{1+1}}{4+4(1)} = \boxed{\frac{\sqrt{2}}{8}}$$

Section 1.4: 1, 4, 12, 15, 18

1) $$\lim_{x\to 0} \frac{\cos x}{x+1} = \frac{\lim_{x\to 0} \cos(x)}{\lim_{x\to 0} x+1} = \frac{\cos(0)}{0+1} = \frac{1}{1} = \boxed{1}$$

4) $$\lim_{x\to 0} \frac{3x \tan x}{\sin x}$$ note: plugging $x=0$ in gives $\frac{0}{0}$, lets try to simplify

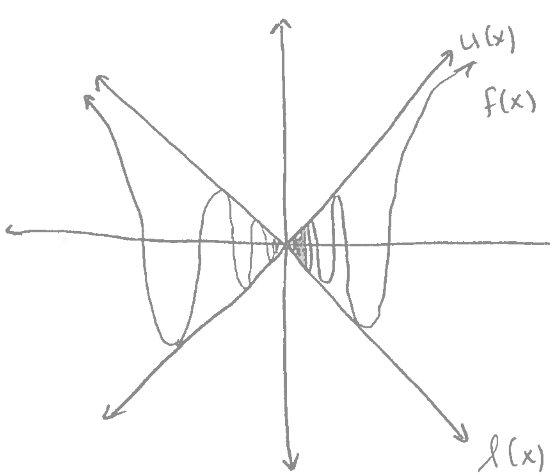
$$= \lim_{x\to 0} \frac{3x\left(\frac{\sin(x)}{\cos(x)}\right)}{\sin(x)} = \lim_{x\to 0} \frac{3x \sin(x)}{\cos(x)\sin(x)} = \lim_{x\to 0} \frac{3x}{\cos(x)}$$

$$= \frac{3(0)}{\cos(0)} = \frac{0}{1} = \boxed{0}$$

12) $$\lim_{t\to 0} \frac{\tan 2t}{\sin 2t - 1}$$ Try plugging in $t=0$ and see what happens

$$= \frac{\tan(2\cdot 0)}{\sin(2\cdot 0)-1} = \frac{\tan(0)}{\sin(0)-1} = \frac{\frac{\sin(0)}{\cos(0)}}{\sin(0)-1} = \frac{\left(\frac{0}{1}\right)}{0-1} = \frac{0}{-1} = \boxed{0}$$

15) $u(x) = |x|$, $f(x) = x \sin(1/x)$, $\ell(x) = -|x|$



since $f(x)$ is trapped between $u(x)$ and $\ell(x)$

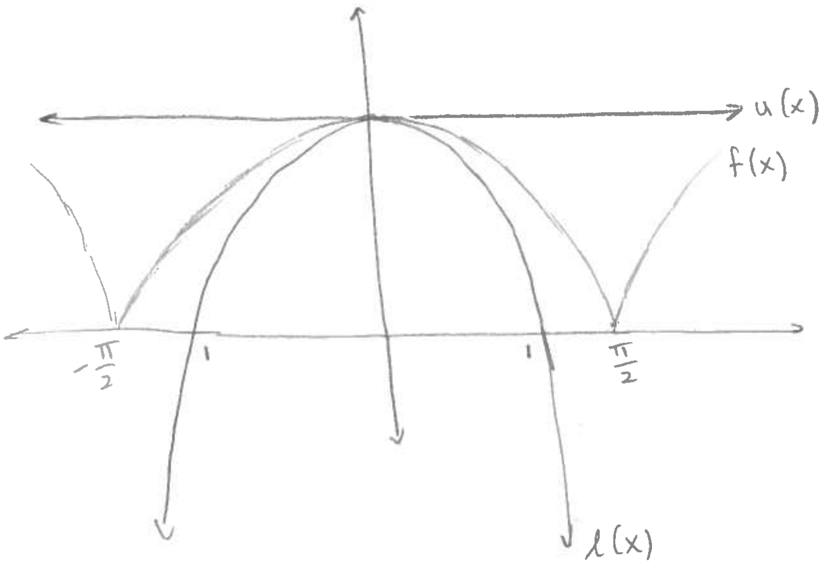
and $\lim_{x\to 0} |x| = 0$

$\lim_{x\to 0} -|x| = 0$

Squeeze Theorem

$\Rightarrow \lim_{x\to 0} x \sin(1/x) = 0$

18) $u(x) = 1, \quad f(x) = \cos^2(x), \quad \ell(x) = 1 - x^2$



since $\ell(x) \le f(x) \le u(x)$

it follows that

$$\lim_{x \to 0} 1 = 1$$

$$\lim_{x \to 0} 1 - x^2 = 1$$

$$\Rightarrow \lim_{x \to 0} \cos^2(x) = 1$$ by squeeze theorem

Section 1.5: 1, 5, 10, 14, 15, 17, 25, 33, 37

1) $$\lim_{x\to\infty} \frac{x}{x-5} = \lim_{x\to\infty} \frac{x}{x-5} \cdot \frac{1/x}{1/x} = \lim_{x\to\infty} \frac{x\left(\frac{1}{x}\right)}{x\left(\frac{1}{x}\right) - 5\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{1}{1 - 5\left(\frac{1}{x}\right)} = \frac{1}{1-5(0)} = \boxed{1}$$

5) $$\lim_{x\to\infty} \frac{x^2}{(x-5)(3-x)} = \lim_{x\to\infty} \frac{x^2}{-x^2+3x+5x-15} = \lim_{x\to\infty} \frac{x^2}{-x^2+8x-15}$$

$$= \lim_{x\to\infty} \frac{x^2\left(\frac{1}{x^2}\right)}{\frac{1}{x^2}\left(-x^2+8x-15\right)} = \lim_{x\to\infty} \frac{1}{-1+8\left(\frac{1}{x}\right) - 15\left(\frac{1}{x^2}\right)}$$

$$= \frac{1}{-1+8(0)-15(0)} = \frac{1}{-1} = \boxed{-1}$$

10) $\lim_{\theta \to \infty} \dfrac{\sin^2 \theta}{\theta^2 - 5}$ note: $0 \le \sin^2\theta \le 1$

$$\Rightarrow \lim_{\theta \to \infty} \frac{0}{\theta^2 - 5} \le \lim_{\theta \to \infty} \frac{\sin^2\theta}{\theta^2 - 5} \le \lim_{\theta \to \infty} \frac{1}{\theta^2 - 5}$$

$$\lim_{\theta \to \infty} \frac{0}{\theta^2 - 5} \to \frac{0}{\infty} = 0$$

$$\lim_{\theta \to \infty} \frac{1}{\theta^2 - 5} = \lim_{\theta \to \infty} \frac{1 \cdot \frac{1}{\theta^2}}{(\theta^2 - 5)\frac{1}{\theta^2}} = \lim_{\theta \to \infty} \frac{(1/\theta^2)}{1 - 5/\theta^2} = \frac{0}{1 - 0} = 0$$

$\Rightarrow$ By squeeze Theorem $\lim_{\theta \to \infty} \dfrac{\sin^2\theta}{\theta^2 - 5} = \boxed{0}$

14) $$\lim_{x \to \infty} \sqrt{\frac{x^2 + x + 3}{(x-1)(x+1)}} = \sqrt{\lim_{x \to \infty} \frac{x^2 + x + 3}{x^2 - 1}}$$

$$= \left[\lim_{x \to \infty} \frac{(x^2 + x + 3)\frac{1}{x^2}}{(x^2 - 1)\frac{1}{x^2}}\right]^{1/2} = \left[\lim_{x \to \infty} \frac{(1 + \frac{1}{x} + \frac{3}{x^2})}{(1 - \frac{1}{x^2})}\right]^{1/2}$$

$$= \left[\frac{1 + 0 + 0}{1 - 0}\right]^{1/2} = \sqrt{1} = \boxed{1}$$

15) $$\lim_{n \to \infty} \frac{n}{2n + 1} = \lim_{n \to \infty} \frac{n \cdot \frac{1}{n}}{(2n+1)\frac{1}{n}} = \lim_{n \to \infty} \frac{1}{2 + \frac{1}{n}}$$

$$= \frac{1}{2 + 0} = \boxed{\frac{1}{2}}$$

17) $\lim\limits_{n\to\infty} \dfrac{n^2}{n+1} = \lim\limits_{n\to\infty} \dfrac{n^2 \cdot \frac{1}{n}}{(n+1)\frac{1}{n}} = \lim\limits_{n\to\infty} \dfrac{n}{1+\frac{1}{n}}$

$\longrightarrow \dfrac{\infty}{1+0} = \boxed{\infty}$

25) $\lim\limits_{n\to\infty} \dfrac{n}{\sqrt{n^2+1}} = \lim\limits_{n\to\infty} \dfrac{n \cdot \frac{1}{n}}{\left(\sqrt{n^2+1}\right)\cdot\frac{1}{n}}$ note: $a\sqrt{b} = \sqrt{a^2 b}$

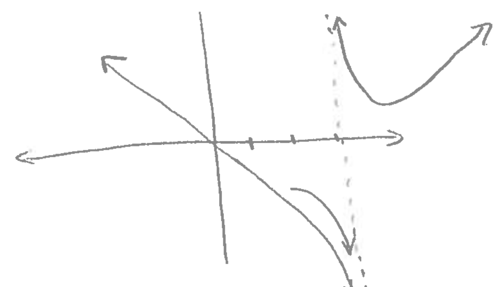
$= \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{\left(\frac{1}{n}\right)^2(n^2+1)}} = \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{1+\frac{1}{n^2}}}$

$= \dfrac{1}{\sqrt{1+0}} = \dfrac{1}{1} = \boxed{1}$

33) $\lim\limits_{x\to 3^-} \dfrac{x^3}{x-3}$ note: at $x=3$ we have a root of the denominator as 3 is not a root of x^3 this means at 3 there is a vertical asymptote

in the $\lim\limits_{x\to 3^-}$ x^3 is positive and $x-3$ is negative

$\Rightarrow \lim\limits_{x\to 3^-} \dfrac{x^3}{x-3} = \boxed{-\infty}$



37) $\lim\limits_{x\to 0^+} \dfrac{[[x]]}{x}$ note $\lim\limits_{x\to 0^+} [[x]] = 0$, $\lim\limits_{x\to 0^+} x = 0$ $\Rightarrow$ we have $\frac{0}{0}$

let us check a progression of points:

$\lim\limits_{x\to 0.1} \dfrac{[[x]]}{x} = \dfrac{0}{0.1} = 0$, $\lim\limits_{x\to 0.01} \dfrac{[[x]]}{x} = \dfrac{0}{0.01} = 0$

$\lim\limits_{x\to 10^{-10}} \dfrac{[[x]]}{x} = \dfrac{0}{10^{-10}} = 0$ $\Rightarrow$ it seems like $\lim\limits_{x\to 0} \dfrac{[[x]]}{x} = \boxed{0}$