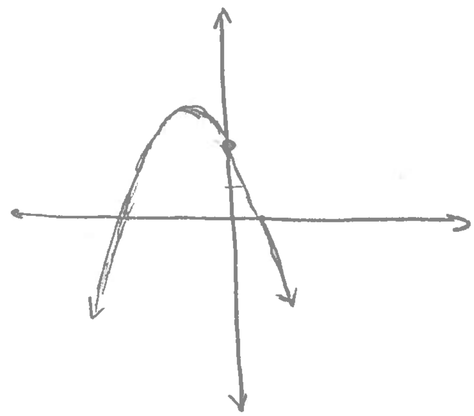
Homework 1 Solutions

Chapter 0 Section 4: 11, 13, 16

11) $y = -x^2 - 2x + 2$

$= -(x^2 + 2x - 2)$



check for x-intercepts (roots)

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(2)}}{2(-1)}$$

$$= \frac{2 \pm \sqrt{4+8}}{-2} = \frac{2 \pm \sqrt{12}}{-2}$$

roots are: $x_1 = -1 + \sqrt{3}$

$x_2 = -1 - \sqrt{3}$

y-intercept: $y = -(0)^2 - 2(0) + 2)$

$y = 2$

13) $x^2 - y^2 = 4$

x-intercepts: $x^2 - (0)^2 = 4$

$x^2 = 4$

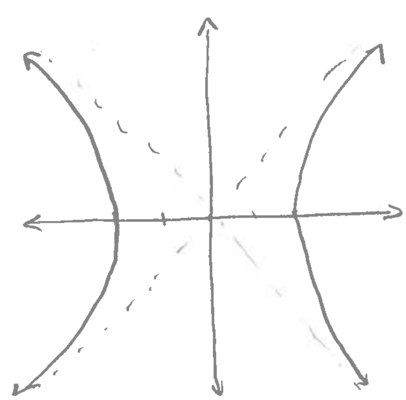
$x = \pm 2$

y-intercepts: $(0)^2 - y^2 = 4$

$y^2 = -4$

no real solutions

x	y
-3	$\pm\sqrt{5}$
-2	0
-1	imaginary
1	imaginary
2	0
3	$\pm\sqrt{5}$



16) $x^2 - 4x + 3y^2 = -2$

x	y
-4	$3y^2 = -34$ imaginary
-2	imaginary
0	imaginary
2	$3y^2 = 2 \quad y = \pm\sqrt{\frac{2}{3}}$
1	$y^2 = \frac{1}{3} \quad y = \pm\sqrt{\frac{1}{3}}$
3	$y^2 = \frac{1}{3} \quad y = \pm\sqrt{\frac{1}{3}}$
4	imaginary

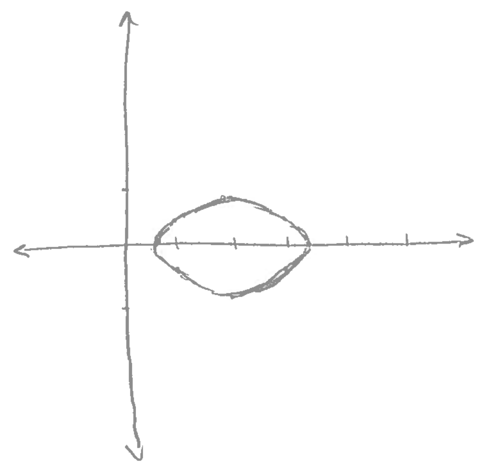
x-intercepts: $x^2 - 4x + 3(0)^2 = -2$

$$x^2 - 4x + 2 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$x_1 = 2 - \sqrt{2} \qquad x_2 = 2 + \sqrt{2}$



y-intercepts: $(0)^2 - 4(0) + 3y^2 = -2$

$$3y^2 = -2$$

$$y^2 = \frac{-2}{3}$$

$\Rightarrow$ no real y-intercepts

Chapter 0 Section 5: 3, 7, 14, 21

3) $G(y) = \dfrac{1}{y-1}$

a) $G(0) = \dfrac{1}{(0)-1} = \dfrac{1}{-1} = -1$

b) $G(0.999) = \dfrac{1}{(.999)-1} = \dfrac{1}{-.001}$

$$= \frac{1}{\frac{-1}{1000}} = -1000$$

c) $G(1.01) = \dfrac{1}{(1.01)-1} = \dfrac{1}{.01}$

$$= \frac{1}{\frac{1}{100}} = 100$$

d) $G(y^2) = \dfrac{1}{(y^2)-1} = \dfrac{1}{y^2 - 1}$

e) $G(-x) = \dfrac{1}{(-x)-1} = \dfrac{1}{-x-1} = \dfrac{-1}{x+1}$

f) $G\left(\frac{1}{x^2}\right) = \dfrac{1}{\frac{1}{x^2} - 1} = \dfrac{1}{\frac{1}{x^2} - \frac{x^2}{x^2}}$

$$= \frac{x^2}{1 - x^2}$$

7) a) $x^2 + y^2 = 1$

for $x = 0$ we have $y^2 = 1 \Rightarrow y = \pm 1$

$\Rightarrow$ we have 2 outputs for 1 input $\Rightarrow$ <u>not a function</u>

b) $xy + y + x = 1 \;,\; x \neq -1$

$$y(x+1) + x = 1$$
$$y(x+1) = 1 - x$$
$$y = \frac{1-x}{x+1}$$

This is a rational <u>function</u>, every input, x gives one output, y, when $x \neq -1$.

c) $x = \sqrt{2y+1}$

$$\Rightarrow x^2 = 2y + 1$$
$$2y = x^2 - 1$$
$$y = \frac{1}{2}x^2 - \frac{1}{2}$$

This is a quadratic polynomial which is a <u>function</u>

d) $x = \dfrac{y}{y+1}$

$$\Rightarrow x(y+1) = y$$
$$xy + x = y$$
$$xy - y = -x$$
$$y(x-1) = -x$$
$$y = \frac{-x}{x-1}$$

Is a rational <u>function</u> provided that $x \neq 1$ $(y \neq -1)$

14) Find the natural (implied) domain

a) $f(x) = \dfrac{4-x^2}{x^2-x-6} = \dfrac{(2-x)(2+x)}{(x-3)(x+2)}$

The denominator has roots at $x=3$, $x=-2$

These are locations where we divide by zero

Domain $= \mathbb{R} - \{-2, 3\}$

b) $G(y) = \sqrt{(y+1)^{-1}} = \sqrt{\dfrac{1}{y+1}}$

Divide by 0: $y=-1$

square root of negative number: $y<-1$

Domain $= \mathbb{R} - (-\infty, -1] = (-1, \infty)$

c) $\phi(u) = |2u+3|$

Divide by zero nowhere

even root of a negatie number nowhere

Domain $= \mathbb{R}$

d) $F(t) = t^{2/3} - 4 = \sqrt[3]{t^2} - 4$

Divide by zero nowhere

even root of a negative number nowhere

Domain $= \mathbb{R}$

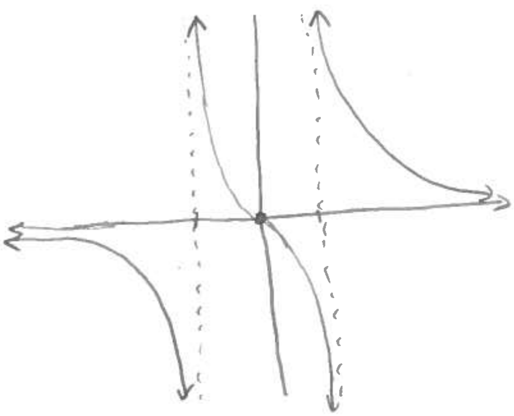
21) $g(x) = \dfrac{x}{x^2-1}$

$g(-x) = \dfrac{-x}{(-x)^2-1} = \dfrac{-x}{x^2-1}$ $\Rightarrow$ $g(-x) = -g(x)$

$g(x)$ is an odd function

$-g(x) = \dfrac{-x}{x^2-1}$

$g(x) = \dfrac{x}{x^2-1} = \dfrac{x}{(x+1)(x-1)}$



x	y
0	0
$\frac{1}{2}$	$\dfrac{\frac{1}{2}}{\frac{1}{4}-1} = \dfrac{1/2}{-3/4} = \dfrac{-2}{3}$
$-\frac{1}{2}$	$\dfrac{-1/2}{-3/4} = \dfrac{2}{3}$
2	$\frac{2}{3}$
-2	$\frac{-2}{3}$

$\Rightarrow$ root at $x=0$

verticle asymptotes at

$x=-1$, $x=1$

Chapter 0 Section 6: 5, 17

5) $f(s) = \sqrt{s^2-4}$

$g(w) = |1+w|$

$(f \circ g)(x) = f(g(x)) = f(|1+x|)$

$= \sqrt{|1+x|^2 - 4}$

$= \sqrt{(1+x)^2-4} = \sqrt{x^2+2x-3}$

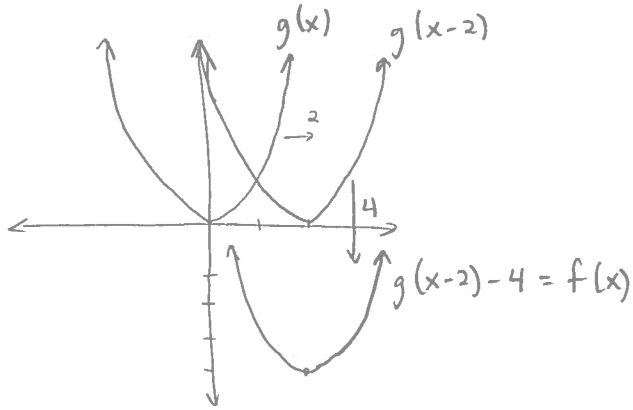
$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x^2-4}\right)$

$= \left|1+\sqrt{x^2-4}\right|$

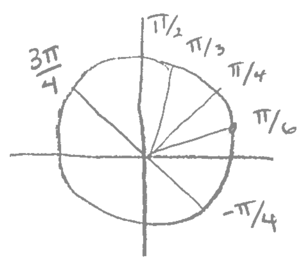
17) $f(x) = (x-2)^2 - 4$

start with $g(x) = x^2$ then $f(x) = g(x-2) - 4$

shift the graph of $g(x)$ 2 to the right, and then 4 down



Chapter 0 Section 7:



9) a) $\tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

b) $\sec \pi = \frac{1}{\cos \pi} = \frac{1}{-1} = \boxed{-1}$

c) $\sec \frac{3\pi}{4} = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = \frac{1}{\frac{-\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

d) $\csc \frac{\pi}{2} = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = \frac{1}{1} = \boxed{1}$

e) $\cot \frac{\pi}{4} = \frac{\cos\left(\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \boxed{1}$

f) $\tan\left(-\frac{\pi}{4}\right) = \frac{\sin(-\pi/4)}{\cos(-\pi/4)} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{-1}$

13) a) $\dfrac{\sin u}{\csc u} + \dfrac{\cos u}{\sec u} = 1$

$\Rightarrow \dfrac{\sin u}{\frac{1}{\sin u}} + \dfrac{\cos u}{\frac{1}{\cos u}} = \sin^2 u + \cos^2 u = 1$ ✓

$\sin^2 x + \cos^2 x = 1$
$\Rightarrow \sin^2 x = 1 - \cos^2 x$

b) $(1 - \cos^2 x)(1 + \cot^2 x) = 1$

$(1 - \cos^2 x)\left(1 + \dfrac{\cos^2(x)}{\sin^2(x)}\right) = [\sin^2(x)]\left[\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}\right]$

$= \dfrac{\sin^2(x)}{\sin^2(x)} \cdot 1 = 1$ ✓

c) $\sin t\,(\csc t - \sin t) = \cos^2(t)$

$\sin t\,(\csc t - \sin t) = \sin t\left(\dfrac{1}{\sin t} - \sin t\right)$

$= \dfrac{\sin t}{\sin t} - \sin^2 t = 1 - \sin^2 t = \cos^2 t$ ✓

d) $\dfrac{1 - \csc^2 t}{\csc^2 t} = \dfrac{-1}{\sec^2 t}$

$\dfrac{1 - \csc^2 t}{\csc^2 t} = \dfrac{1 - \frac{1}{\sin^2 t}}{\frac{1}{\sin^2 t}} = \dfrac{\frac{\sin^2 t - 1}{\sin^2 t}}{\frac{1}{\sin^2 t}} = -(1 - \sin^2 t)$

$= -\cos^2 t = \dfrac{-1}{\frac{1}{\cos^2 t}} = \dfrac{-1}{\sec^2 t}$ ✓

26) a) $f(t) = \cot(t) + \sin(t)$

$f(-t) = \cot(-t) + \sin(-t)$

$= \dfrac{\cos(-t)}{\sin(-t)} + \sin(-t) = \dfrac{\cos t}{-\sin t} - \sin(t) = -\cot(t) - \sin(t)$

$-f(t) = -\cot(t) - \sin(t)$

$\Rightarrow f(-t) = -f(t)$, $f(t)$ is <u>odd</u>

b) $\sin^3 t = f(t)$

$f(-t) = \sin^3(-t) = [\sin(-t)]^3 = [-\sin(t)]^3 = -\sin^3(t)$

$-f(t) = -\sin^3(t)$

$\Rightarrow f(-t) = -f(t) \Rightarrow f(t)$ is <u>odd</u>

c) $\sec t = f(t)$

$f(-t) = \sec(-t) = \dfrac{1}{\cos(-t)} = \dfrac{1}{\cos(t)} = \sec(t)$

$-f(t) = -\sec(t)$

$\Rightarrow f(t) = f(-t) \Rightarrow f(t)$ is <u>even</u>

d) $\sqrt{\sin^4(t)} = f(t) = \sin^2(t)$

$f(-t) = \sin^2(-t) = [\sin(-t)]^2 = [-\sin(t)]^2 = \sin^2(t)$

$\Rightarrow f(t) = f(-t) \Rightarrow f(t)$ is <u>even</u>

e) $\cos(\sin(t)) = f(t)$

$f(-t) = \cos(\sin(-t)) = \cos(-\sin(t)) = \cos(\sin(t))$

$\Rightarrow f(t) = f(-t) \Rightarrow f(t)$ is <u>even</u>

f) $x^2 + \sin x = f(x)$

$f(-x) = (-x)^2 + \sin(-x) = x^2 - \sin(x)$

$-f(x) = -x^2 - \sin(x)$

$\Rightarrow \quad f(x) \neq f(-x)$ and $f(-x) \neq -f(x) \Rightarrow f(x)$ is <u>neither</u>

Chapter 1 Section 1:

3) $\lim\limits_{x \to -2} x^2 + 2x - 1 = \lim\limits_{x \to -2} (x)^2 + \lim\limits_{x \to -2} 2x + \lim\limits_{x \to -2} -1$

$= 4 - 4 - 1 = \boxed{-1}$

4) $\lim\limits_{x \to -2} x^2 + 2t - 1 = \lim\limits_{x \to -2} x^2 + \lim\limits_{x \to -2} 2t + \lim\limits_{x \to -2} -1$

$= 4 + 2t - 1 = \boxed{3 + 2t}$

13) $\lim\limits_{t \to 2} \dfrac{\sqrt{(t+4)(t-2)^4}}{(3t-6)^2} = \lim\limits_{t \to 2} \dfrac{\sqrt{t+4}\,\sqrt{(t-2)^4}}{(3(t-2))^2} = \lim\limits_{t \to 2} \dfrac{\sqrt{t+4}\,(t-2)^2}{9(t-2)^2}$

in the sense of the limit we can cancel $= \lim\limits_{t \to 2} \dfrac{\sqrt{t+4}}{9} = \dfrac{\sqrt{6}}{9}$

14) $\lim\limits_{t \to 7^+} \dfrac{\sqrt{(t-7)^3}}{t-7} = \lim\limits_{t \to 7^+} \dfrac{(t-7)\sqrt{t-7}}{(t-7)} = \lim\limits_{t \to 7^+} \sqrt{t-7} = \sqrt{0} = \boxed{0}$

note: $\lim\limits_{t \to 7}$ and $\lim\limits_{t \to 7^-}$ of $\dfrac{\sqrt{(t-7)^3}}{(t-7)}$ do <u>Not Exist</u>

17) $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h} = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h}$

$= \lim_{h \to 0} \frac{4h + h^2}{h} = \lim_{h \to 0} 4 + h = \boxed{4}$

18) $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$

$= \lim_{h \to 0} 2x + h = \boxed{2x}$