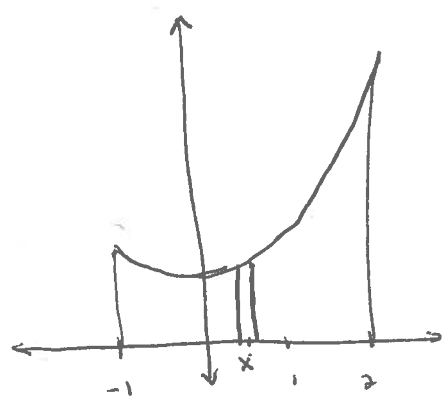
HW 10 Solutions

5.1: 1, 3, 6, 13, 19, 20

1)



$$\Delta A \approx \sum \overset{\text{height}}{f(x)}\, \overset{\text{width}}{\Delta x}$$

$$A = \int_{-1}^{2} f(x)\,dx = \int_{-1}^{2} (x^2+1)\,dx$$

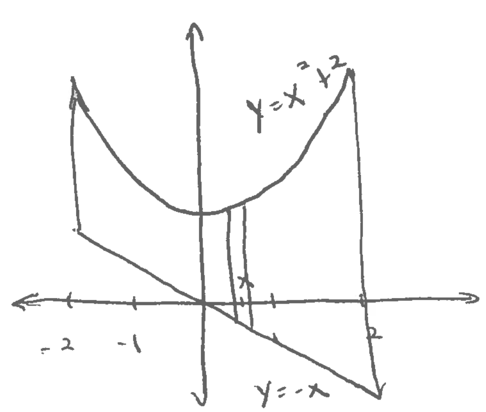
$$= \left[\frac{1}{3}x^3 + x\right]_{-1}^{2}$$

$$= \left(\frac{1}{3}(2)^3 + 2\right) - \left(\frac{1}{3}(-1)^3 + (-1)\right)$$

$$= \frac{8}{3} + 2 - \left(-\frac{1}{3} - 1\right)$$

$$= \frac{8}{3} + \frac{1}{3} + 2 + 1 = \boxed{6}$$

3)



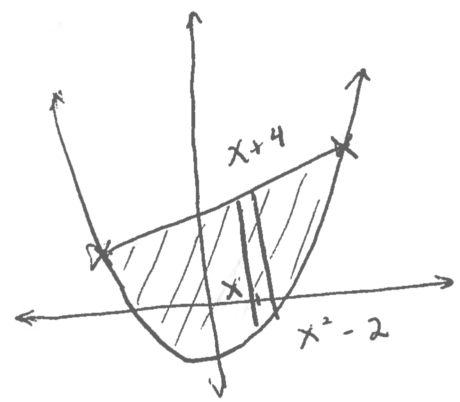
$$\Delta A \approx \sum \Big[\underbrace{\overset{\text{top}}{(x^2+2)} - \overset{\text{bottom}}{(-x)}}_{\text{height}}\Big] \underbrace{\Delta x}_{\text{width}}$$

$$A = \int_{-2}^{2} (x^2 + x + 2)\,dx$$

$$= \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right]_{-2}^{2} = \left(\frac{1}{3}(2)^3 + \cancel{\frac{1}{2}(2)^2} + 2(2)\right) - \left(\frac{1}{3}(-2)^3 + \cancel{\frac{1}{2}(-2)^2} + 2(-2)\right)$$

$$= \frac{2}{3}\cdot 8 + 8 = \frac{16}{3} + 8 = \boxed{\frac{40}{3}}$$

6)



$\Delta A \approx \sum \left[(x+4)-(x^2-2)\right]\Delta x$ ②

$$A = \int_{-1}^{2} (-x^2 + x + 6)\,dx$$

$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-1}^{2}$$

Intersections $\quad x + 4 = x^2 - 2$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = -1,\ x = 2$$

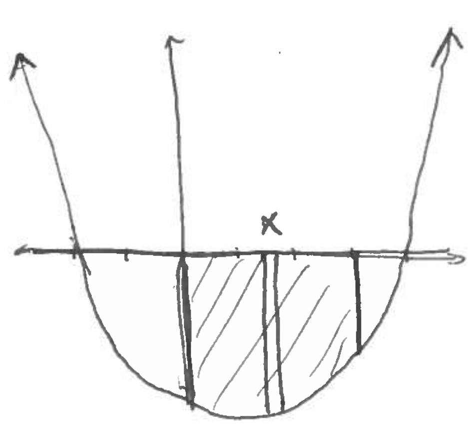
$$A = \left(-\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 6(2)\right) - \left(\frac{-1}{3}(-1)^3 + \frac{(-1)^2}{2} + 6(-1)\right)$$

$$= \left(-\frac{8}{3} + 2 + 12\right) - \left(\frac{1}{3} + \frac{1}{2} - 6\right)$$

$$= -\frac{9}{3} + 20 - \frac{1}{2} = 17 - \frac{1}{2} = \boxed{\frac{33}{2}}$$

13) $y = (x-4)(x+2)$, $y = 0$ between $x = 0$, $x = 3$

$\Delta A = \sum\left(0 - (x-4)(x+2)\right)\Delta x$



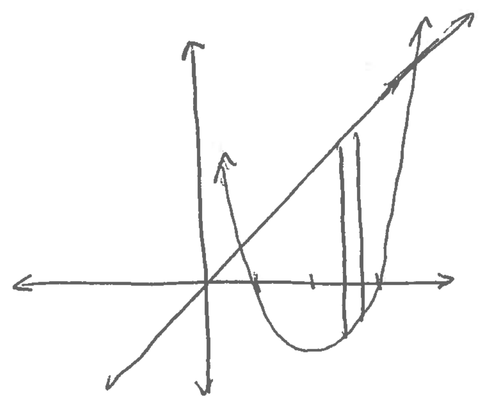
$$A = \int_{0}^{3} (-x^2 + 2x + 8)\,dx$$

$$= \left[-\frac{1}{3}x^3 + x^2 + 8x\right]_0^3$$

$$= \left(-\frac{1}{3}(3)^3 + 3^2 + 8(3)\right) = -9 + 9 + 24$$

$$= \boxed{24}$$

19) $y = (x-3)(x-1)$, $y = x$



$$\Delta A \approx \sum \left(x - (x^2 - 4x + 3)\right)\Delta x$$

$$A = \int_{\frac{5-\sqrt{13}}{2}}^{\frac{5+\sqrt{13}}{2}} (-x^2 + 5x - 3)\,dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{5}{2}x^2 - 3x\right]_{\frac{5-\sqrt{13}}{2}}^{\frac{5+\sqrt{13}}{2}}$$

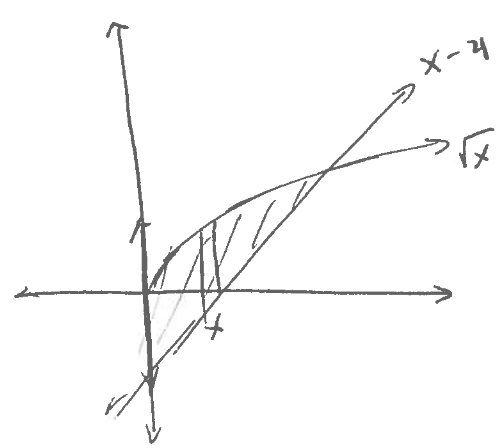
$$= \boxed{\frac{13\sqrt{13}}{6}}$$ (use a calculator)

intersection: $x^2 - 4x + 3 = x$

$x^2 - 5x + 3 = 0$

$x = \dfrac{5 \pm \sqrt{25 - 4(1)(3)}}{2} = \dfrac{5 \pm \sqrt{13}}{2}$

20) $y = \sqrt{x}$, $y = x - 4$, $x = 0$



$$\Delta A \approx \sum \left(\sqrt{x} - (x-4)\right)\Delta x$$

$$A = \int_0^{\frac{9+\sqrt{17}}{2}} (\sqrt{x} - x + 4)\,dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}x^2 + 4x\right]_0^{\frac{9+\sqrt{17}}{2}}$$

$$= \boxed{\frac{121}{12} + \frac{17\sqrt{17}}{12} \approx 16}$$

Intersection

$\sqrt{x} = x - 4$

$x = (x-4)^2 = x^2 - 8x + 16$

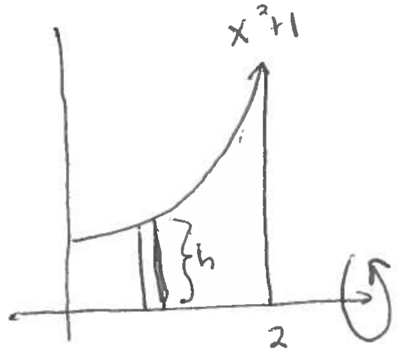
$x^2 - 9x + 16 = 0$

$x = \dfrac{9 \pm \sqrt{81 - 4(16)}}{2} = \dfrac{9 \pm \sqrt{17}}{2}$

$x = \dfrac{9 + \sqrt{17}}{2}$

5.2: 1, 3, 5, 12

1)



$\Rightarrow$ Volume $= \pi r^2$

$$\Delta V \approx \sum \pi (x^2+1)^2 \Delta x$$

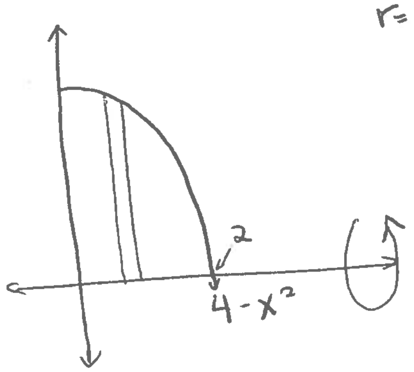
$$V = \int_0^2 \pi (x^2+1)^2 \, dx$$

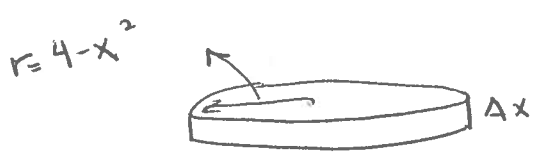
$$= \pi \int_0^2 (x^4 + 2x^2 + 1) \, dx = \pi \left[\frac{1}{5}x^5 + \frac{2}{3}x^3 + x \right]_0^2$$

$$= \pi \left(\frac{1}{5}(2)^5 + \frac{2}{3}(2)^3 + 2 \right) - \pi (0)$$

$$= \pi \left(\frac{32}{5} + \frac{16}{3} + 2 \right) = \boxed{\frac{206\pi}{15}}$$

3) a)





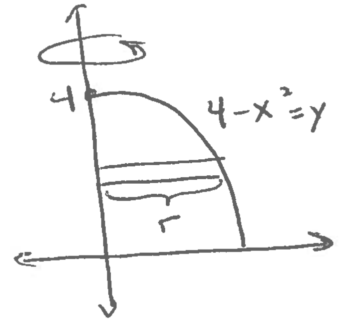
$$\Delta V \approx \sum \pi (4-x^2)^2 \Delta x$$

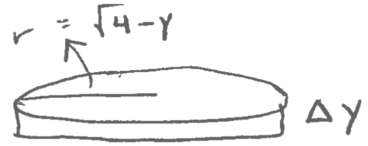
$$V = \int_0^2 \pi (16 - 8x^2 + x^4) \, dx$$

$$= \pi \int_0^2 (x^4 - 8x^2 + 16) \, dx = \pi \left[\frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x \right]_0^2$$

$$= \pi \left(\frac{32}{5} - \frac{64}{3} + 32 \right) = \boxed{\frac{256\pi}{15}}$$

3b)





$\Delta V \approx \pi\left(\sqrt{4-y}\right)^2 \Delta y$

$V = \int_0^4 \pi(4-y)\,dy$

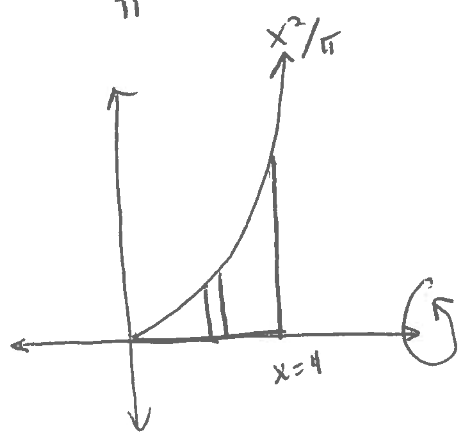
$y = 4 - x^2 \Rightarrow x^2 = 4 - y$

$x = \sqrt{4-y} = r$

$V = \pi\left[4y - \frac{1}{2}y^2\right]_0^4 = \pi\left(4(4) - \frac{1}{2}(4)^2\right)$

$= \pi(16-8) = \boxed{8\pi}$

5) $y = \frac{x^2}{\pi}$, $x = 4$, $y = 0$

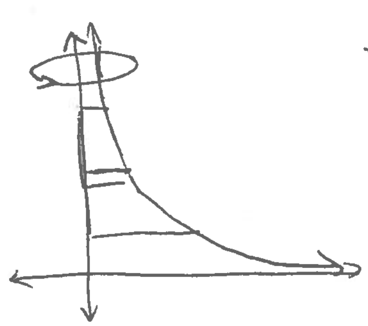


$\Delta V \approx \sum \pi\left(\frac{x^2}{\pi}\right)^2 \Delta x$

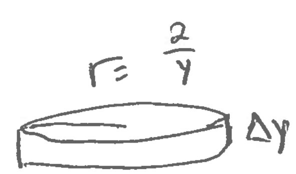
$V = \int_0^4 \frac{1}{\pi} x^4\,dx = \frac{1}{\pi}\left[\frac{1}{5}x^5\right]_0^4$

$= \frac{4^5}{5\pi} = \boxed{\frac{1024}{5\pi}}$

12) $x = \frac{2}{y}$, $y = 2$, $y = 6$, $x = 0$



$\Rightarrow$



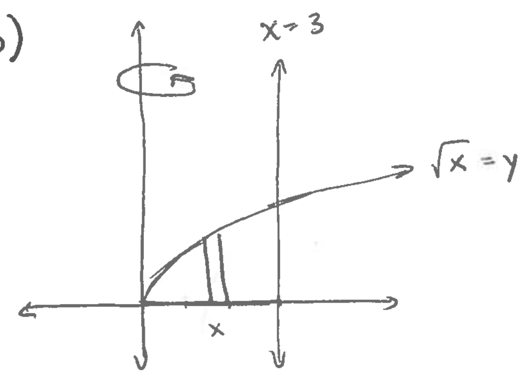
$\Delta V \approx \sum \pi\left(\frac{2}{y}\right)^2 dy$

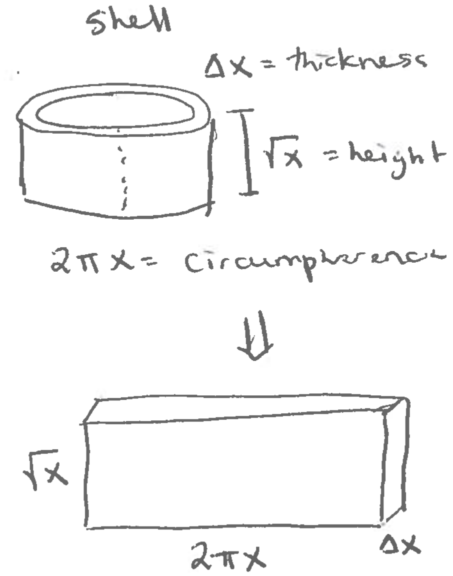
$V = \pi\int_2^6 4y^{-2}\,dy = \pi\left[-4y^{-1}\right]_2^6$

$= -4\pi\left(\frac{1}{6} - \frac{1}{2}\right) = \boxed{\frac{4\pi}{3}}$

5.3: 3, 6, 7

3)



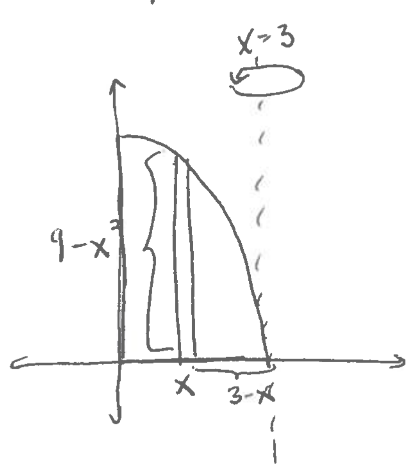


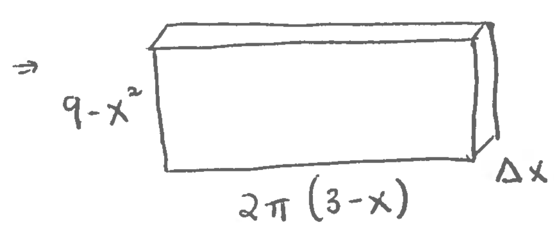
$$\Delta V \approx \sum 2\pi x\sqrt{x}\,\Delta x$$

$$V = \int_0^3 2\pi x\sqrt{x}\,dx = 2\pi\int_0^3 x^{3/2}\,dx = 2\pi\left[\frac{2}{5}x^{5/2}\right]_0^3$$

$$= \frac{4\pi}{5}(3)^{5/2} = \frac{4\pi}{5}\cdot 9\sqrt{3} = \boxed{\frac{36\sqrt{3}\,\pi}{5}}$$

6) $y = 9-x^2$, $x=0$, $y=0$, about $x=3$





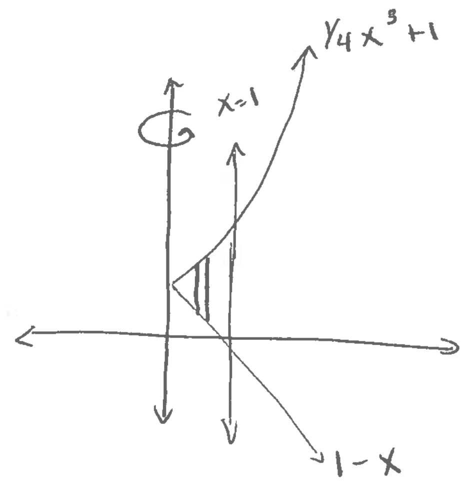
$$\Delta V = \sum 2\pi(3-x)(9-x^2)\,\Delta x$$

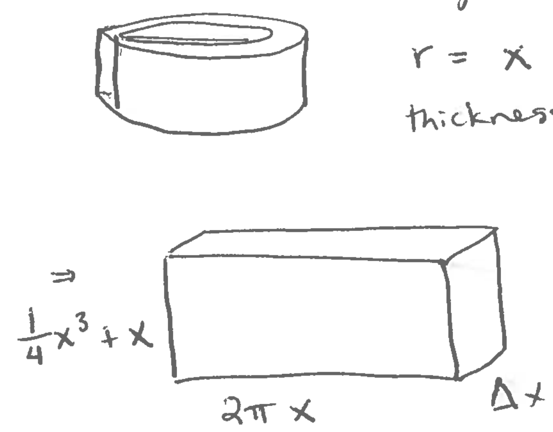
$$V = \int_0^3 2\pi\left(27-9x-3x^2+x^3\right)dx$$

$$= 2\pi\left[27x - \frac{9}{2}x^2 - x^3 + \frac{1}{4}x^4\right]_0^3$$

$$= 2\pi\left(81 - \frac{81}{2} - 27 + \frac{81}{4}\right) = \boxed{\frac{135\pi}{2}}$$

7) $y = \frac{1}{4}x^3 + 1, \quad y = 1 - x, \quad x = 1$ about y-axis





height $= \left(\frac{1}{4}x^3 + 1\right) - (1 - x)$

$r = x$

thickness $= \Delta x$

$$\Delta V \approx \sum 2\pi x \left(\frac{1}{4}x^3 + x\right)\Delta x$$

$$V = \int_0^1 2\pi x \left(\frac{1}{4}x^3 + x\right) dx = 2\pi \int_0^1 \left(\frac{1}{4}x^4 + x^2\right) dx$$

$$= 2\pi \left[\frac{1}{20}x^5 + \frac{1}{3}x^3\right]_0^1$$

$$= 2\pi\left(\frac{1}{20} + \frac{1}{3}\right) = 2\pi\left(\frac{3}{60} + \frac{20}{60}\right)$$

$$= 2\pi\left(\frac{23}{60}\right) = \frac{46\pi}{60} = \boxed{\frac{23\pi}{30}}$$

5.4 : 1, 3, 7, 8

1) $y = 4x^{3/2}$ $\quad x = 1/3,\ x = 5$

$$\frac{dy}{dx} = \frac{d}{dx}\left[4x^{3/2}\right] = 6x^{1/2} = 6\sqrt{x}$$

$$L = \int_{1/3}^{5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{1/3}^{5} \sqrt{1 + (6\sqrt{x})^2}\, dx = \int_{1/3}^{5} \sqrt{1 + 36x}\, dx$$

$u = 1 + 36x$

$du = 36\, dx$

$x = \frac{1}{3} \Rightarrow u = 13$

$x = 5 \Rightarrow u = 181$

$$= \frac{1}{36}\int_{u=13}^{u=181} \sqrt{u}\, du = \frac{1}{36}\left[\frac{2}{3}u^{3/2}\right]_{13}^{181}$$

$$= \frac{1}{54}\left((181)^{3/2} - (13)^{3/2}\right)$$

$$= \boxed{\frac{1}{54}\left(181\sqrt{181} - 13\sqrt{13}\right)}$$

3) $y = (4 - x^{2/3})^{3/2}$ $\quad x = 1,\ x = 8$

$$\frac{dy}{dx} = \frac{3}{2}(4 - x^{2/3})^{1/2} \cdot \frac{-2}{3}x^{-1/3}$$

$$= \frac{-\sqrt{4 - x^{2/3}}}{x^{1/3}}$$

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^8 \sqrt{1 + \left(\frac{-\sqrt{4 - x^{2/3}}}{x^{1/3}}\right)^2}\, dx = \int_1^8 \sqrt{1 + \left(\frac{4 - x^{2/3}}{x^{2/3}}\right)}\, dx$$

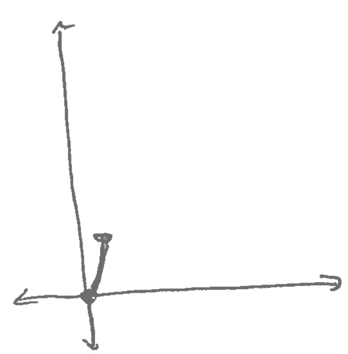
$$= \int_1^8 \sqrt{\cancel{1} + 4x^{-2/3} - \cancel{1}}\, dx = \int_1^8 \sqrt{4x^{-2/3}}\, dx = \int_1^8 2x^{-1/3}\, dx$$

$$= 2\left[\frac{3}{2} x^{2/3}\right]_{x=1}^{8} = 3\left((8)^{2/3} - (1)^{2/3}\right)$$

$$= 3(2^2 - 1)$$

$$= 3 \cdot 3 = \boxed{9}$$

7) $x = \frac{t^3}{3}$ $\quad y = \frac{t^2}{2}$ $\quad 0 \le t \le 1$



t	x	y
0	0	0
$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$
1	$\frac{1}{3}$	$\frac{1}{2}$

$\frac{dx}{dt} = t^2$

$\frac{dy}{dt} = t$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(t^2)^2 + (t)^2}\, dt = \int_0^1 \sqrt{t^4 + t^2}\, dt = \int_0^1 \sqrt{t^2(t^2+1)}\, dt$$

$$L = \int_0^1 t\sqrt{t^2+1}\,dt \qquad u = t^2+1 \qquad t=0 \Rightarrow u=1$$
$$du = 2t\,dt \qquad t=1 \Rightarrow u=2$$

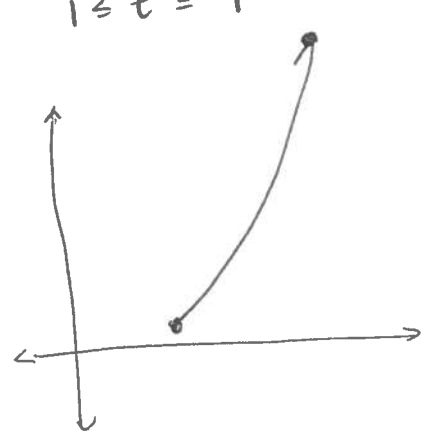
$$= \frac{1}{2}\int_0^1 2t\sqrt{t^2+1}\,dt = \frac{1}{2}\int_1^2 \sqrt{u}\,du$$

$$= \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^2 = \frac{1}{3}\left(2^{3/2} - 1^{3/2}\right)$$

$$= \boxed{\frac{1}{3}\left(2\sqrt{2} - 1\right)}$$

8) $x = 3t^2 + 2$, $y = 2t^3 - \frac{1}{2}$

$1 \le t \le 4$



t	x	y
1	5	$\frac{3}{2}$
2	14	$\frac{31}{2}$
3	29	$\frac{107}{2}$
4	50	$\frac{255}{2}$

$$\frac{dx}{dt} = 9t \qquad \frac{dy}{dt} = 6t^2$$

$$L = \int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int_1^4 \sqrt{(9t)^2 + (6t^2)^2}\,dt = \int_1^4 \sqrt{81t^2 + 36t^4}\,dt$$

$$= \int_1^4 \sqrt{9t^2(9t^2+4)}\,dt = \int_1^4 3t\sqrt{9t^2+4}\,dt$$

11

$$L = \int_1^4 3t\sqrt{9t^2+4}\,dt$$

$u = 9t^2 + 4$

$du = 18t\,dt$

$t = 1 \Rightarrow u = 9 + 4 = 13$

$t = 4 \Rightarrow u = 9 \cdot 16 + 4 = 148$

$$= \frac{1}{6}\int_1^4 18t\sqrt{9t^2+4}\,dt$$

$$= \frac{1}{6}\int_{u=13}^{u=148} \sqrt{u}\,du = \frac{1}{6}\left[\frac{2}{3}u^{3/2}\right]_{13}^{148}$$

$$= \frac{1}{9}\left((148)^{3/2} - (13)^{3/2}\right)$$

$$= \boxed{\frac{1}{9}\left(148\sqrt{148} - 13\sqrt{13}\right)}$$