## 1170:Lab6

October 8th, 2013

## Goals For This Week

In this sixth lab we will explore Optimization and Bumble Bees

- Maximization Problems
- Schemes for Nectar Collection
- Marginal Value Theorem and Maximizing


## 1 Maximization Problems

One application of Calculus arises in Optimization problems, like finding Maximizations and Minimizations. The maximal and minimal values of a function occur at critical points. Critical points of a function are locations where the derivative of the function is equal to zero, or at the endpoints of the interval over which we are exploring the function. A maxima occurs where the 2 nd derivative of the function is negative at a non-endpoint critical point, and a minima occurs when the 2 nd derivative is positive. The global maximum and minimum for the interval are located at critical points, and these are the values which biologically we are usually most interested in.

## 2 Schemes for Nectar Collection

Suppose you are a bee flying from flower to flower collecting nectar. The longer you stay at a flower the more nectar you will collect, but at a slower and slower rate as you deplete the flower. When you are flying to a new flower however you can collect no nectar. Let us define the rate per visit as following

$$
\begin{gather*}
R(t)=\frac{\text { food per visit }}{\text { total time per visit }}  \tag{1}\\
R(t)=\frac{\text { food per visit }}{\text { time on flower }+ \text { travel time }}  \tag{2}\\
R(t)=\frac{\mathrm{F}(\mathrm{t})}{t_{f}+t_{t}} \tag{3}
\end{gather*}
$$

Suppose that $F(t)=\frac{t}{t+0.5}$ and that the travel time is $t_{t}=1$

$$
\begin{equation*}
R(t)=\frac{t}{(t+0.5)(t+1)} \tag{4}
\end{equation*}
$$

What are some strategies that a bee could use to collect nectar?

### 2.1 Strategy 1

Suppose that a bee spends 5 second collecting nectar before travelling to the next flower.
What is the amount of nectar that the bee has collected as a function of time?
For a single flower and single visit this is equal to $\mathrm{F}(\mathrm{t})$
To visualize this let us enter the following:

```
t<-seq(0,5,0.01)
F<-function(t){t/(t+.5)}
plot(t,F(t),type='l')
```

for the next second, $\mathrm{t}=(5,6)$ there is no nectar collected as the bee is travelling to the next flower. to plot this piecewise defined function we will need to use some new commands in R .

- The first command is curve $(a, b, c)$ which will plot a function named a from (b,c).
- The second command is the ifelse command, this will allow us to put conditional statements.

For example:

```
F<-function(t) ifelse(t<=5, t/(t+0.5), 5/5.5)
curve(F,0,6)
```

These commands will define as a Function $F(t)=\frac{t}{t+0.5}$ as long as $t<=5$, otherwise it will be defined as $5 / 5.5$ which is the value of $\mathrm{F}(5)$.

Now the bee will reach another flower an begin harvesting again the bee has collected $F(5)=5 / 5.5=$ .90909 nectar in 6 seconds which is 0.151515 nectar per second.

### 2.2 Strategy 2

Now the bee collects nectar for 2 seconds before travelling.

```
G<-function(t) ifelse(t<=2, t/(t+.5), ifelse(t<=3, 2/2.5, ifelse(t<=5,
+(t-3)/(t-3+.5)+2/2.5, (5-3)/(5-3+0.5)+2/2.5)))
curve(G,0,6)
```

Which strategy allows the bee to gather more honey?
Is there a way to figure out what the bee's best strategy is?

## 3 Marginal Value Theorem

The Marginal value theorem states that the best strategy is to leave when the instantaneous rate is equal to the average rate.

$$
\begin{equation*}
F^{\prime}(t)=\frac{F(t)}{t_{f}+t_{t}}=R(t) \tag{5}
\end{equation*}
$$

For our problem $F(t)=\frac{t}{0.5+t}$

$$
\begin{gather*}
F^{\prime}(t)=\frac{(t+0.5) \frac{d t}{d t}-t \frac{d(t+0.5)}{d t}}{(t+0.5)^{2}}  \tag{6}\\
F^{\prime}(t)=\frac{(t+0.5)-t}{(t+0.5)^{2}} \tag{7}
\end{gather*}
$$

therefore

$$
\begin{equation*}
F^{\prime}(t)=\frac{0.5}{(t+0.5)^{2}} \tag{8}
\end{equation*}
$$

Now we want to set this equal to $\mathrm{R}(\mathrm{t})$ and solve for t . Where $R(t)=\frac{t}{(t+0.5)(t+1)}$

$$
\begin{gather*}
F^{\prime}(t)=\frac{0.5}{(t+0.5)^{2}}=\frac{t}{(t+0.5)(t+1)}=R(t)  \tag{9}\\
0.5(t+1)=t(t+0.5)  \tag{10}\\
t^{2}=0.5 \tag{11}
\end{gather*}
$$

therefore $t=0.707$ is how long the bee should stay on the flower before leaving.
How much nectar would a bee collect using this strategy over the course of 5 seconds?

```
H<-function(t){t/(t+0.5)}
H(0.707)
```

This is how much a bee makes for a single flower, after that they travel for 1 second.
The following is a table of how much the bee makes in total over seconds. Was this strategy more effective?

| time | total time | nectar | total nectar |
| :---: | :---: | :---: | :---: |
| .707 | .707 | .586 | .586 |
| 1 | 1.707 | 0 | .586 |
| .707 | 2.414 | .586 | 1.172 |
| 1 | 3.414 | 0 | 1.172 |
| .707 | 4.121 | .586 | 1.758 |
| 1 | 5.121 | 0 | 1.758 |
| .707 | 5.898 | .586 | 2.344 |
| .102 | 6 | 0 | 2.344 |

## 4 Assignment for this week

Suppose that our bee finds itself in a less populated flower patch. Now instead of 1 second travel time, it takes the bee 5 seconds to travel between flowers. Assume that $F(t)=\frac{t}{t+0.5}$ remains the same.

1. How long should the bee now stay at the flower to maximize its nectar collection (show your work)?
2. Include with your solution a plot of the total nectar collected by the bee as a function of time as it visits 2 flowers using the commands we learned to plot piecewise defined functions. Hint: The total time of the plot should be 2 times your solution to the first part plus 5 .
