

1170:Lab4

September 24th, 2013

Goals For This Week

In this fourth lab we will explore using the graph of the Derivative of a function to gain insight into the behavior of the function.

- Plotting the Derivative of a function
- Changing Domains to zoom in on features
- Functional Composition (to prepare for Midterm 1)

1 Plotting The Derivative

The Derivative of a function is also a function. As such we can graph the derivative in the same way that we would graph the original function, we need to define the function in R and create a domain sequence.

For example consider the function: $f(x) = x^3 - 4x^2 + e^x$

This function has the derivative: $f'(x) = 3x^2 - 8x + e^x$

For convenience lets call $f'(x)=g(x)$. Then to graph $g(x)$ in R we can enter the following commands

```
> g<-function(x){3*x^(2)-8*x+exp(x)}  
> x<-seq(0,5,.01)  
> plot(x,g(x),type='l')
```

Now the graph of $g(x)$ appears to start slightly above $y=0$ when $x=0$, then dip below the x -axis before rising above the x -axis again before $x=2$. To better see this though let us add the x -axis into our plot with the following command.

```
> lines(x,x*0)
```

Now keeping in mind that this is the plot of the derivative of $f(x)$, what can we say about f ?

- When is $f(x)$ increasing?
- When is $f(x)$ decreasing?
- When does $f(x)$ have relative minima and maxima?
- What are the critical points of $f(x)$?

1.1 When is $f(x)$ increasing?

A function is increasing when its derivative is positive. For example consider distance vs. velocity. Velocity is the derivative of distance, we then move forward (or increase in distance) when our velocity is positive.

For our problem we are going to be increasing when $g(x) > 0$. Looking at our plot it isn't entirely clear when that is happening, or that in the domain we have chosen so far includes all times when this may be happening.

First let us expand our domain to be sure that we aren't missing any interesting behavior of this function.

```
> fulldomain<-seq(-10,10,.01)
> plot(fulldomain,g(fulldomain),type='l')
> lines(fulldomain,fulldomain*0)
```

It now appears as though all the action was happening between $x=0$ and $x=2$. So now lets zoom in on them

```
> intdom<-seq(0,2,.001)
> plot(intdom,g(intdom),type='l')
> lines(intdom,intdom*0)
```

We can now see that $g(x) > 0$ when $x < 0.2$ or $x > 1.7$. Those are the regions of the domain where $f(x)$ is increasing.

1.2 When is $f(x)$ decreasing?

A function is decreasing when its derivative is negative. For example consider again distance vs. velocity. We are going to move backwards (or decrease in distance) when our velocity is negative.

For our problem then, our function $f(x)$ is going to be decreasing when $g(x) < 0$. Looking at our zoomed in plot we see that this is happening when $0.2 < x < 1.7$

1.3 When does $f(x)$ have relative minima and maxima?

A relative minima or maxima will occur when the derivative is equal to 0, ($g(x)=0$). To see this consider the ball being thrown in the air problem from lab 2. The ball reached its maximum height, relative maxima, when the tangent line was horizontal, or in other words when the derivative was equal to 0.

How can we tell whether a point where $g(x)=0$ is a relative minima or maxima of the function $f(x)$?

A relative maxima will have the feature that we were increasing in value before the maxima, and then decreasing in value after the maxima. This relates to the derivative function, $g(x)$ being positive before (or to the left of the maxima) and then negative after (or to the right of the maxima).

The opposite is true for a relative minima. A relative minima is characterized by the derivative being negative to left of the point and then positive to the right of the point.

For our example problem this means that we have 2 points of interest, x is approximately 0.2, and x is approximately 1.7

The point $x=0.2$ is a relative maxima, because $g(x)$ was positive to the left of $x=0.2$ and negative to the right.

Similarly, $x=1.7$ is a relative minima.

1.4 What are the critical points of $f(x)$?

Critical points of a function are endpoints of the domain, and locations where the derivative function are equal to 0. For this problem since we were not given any special domain we don't have any endpoints. As such our only critical values are $x=0.2$, $x=1.7$

2 Functional Composition

Sometimes, especially in lab work we do not have a direct way to measure a quantity that you would like directly. For example Viscosity is a physical property of a fluid which is difficult to directly measure. However for certain fluids like water the relationship between temperature and viscosity is well categorized.

Let us define this function as $V(T) = 2.414 * 10^{-5} * 10^{(247.8/(T - 140))}$

Our Lab also has a thermal bath capable of raising the temperature of the water in it at a steady rate of 1 degree per minute until it reaches the boiling point. Assuming that the water starts at an initial temperature of 150 degrees Kelvin, and we heat the water in the thermal bath for 1 hour can we describe how the viscosity of the water changes as a function of time?

First we need to interpret the problem. We have 2 functions, temperature as a function of time, and viscosity as a function of temperature. Let us define these in R:

```
Temp<-function(time){150+time}
Viscosity<-function(Temp){2.414*10^(-5)*10^(247.8/(Temp-140))}
time<-seq(0,60,.01)
plot(time,Viscosity(Temp(time)),type='l')
```

Check your answer by hand by doing the functional composition and then plotting the resulting function by hand.

3 Assignment for this week

In dire need of a vehicle and short of funds you visited a less than reputable dealership. With no other options you bought the only remaining car on the lot. The engine is a little old and made of a weak alloy that swells and shrinks with the temperature. This causes gaps in the pistons which causes the gas to leak and causes the car to lose power and speed as the temperature increases. After months of collecting data you find that the cars maximum velocity of a function of the temperature behaves as the following function:

$$V(temp) = 80 * \exp(-(temp/80)^2)$$

On the 24 hour road trip you want to take you find that the temperature is going to behave like a sin wave.

$$temp(time) = 80 + 12 * \sin(2 * \pi * time/24)$$

Plot the velocity of the car as a function of time for the full 24 hour trip. What can you say about the distance that the car has travelled, using the example problem as a guide?