## 1170:Lab2

September 10th, 2013

## Goals For This Week

In this second lab we will explore using graphs to get insight into some features of functions.

- Use log plots to visualize exponential functions and other rapidly growing functions
- Using secant lines to approximate tangent lines and therefore the derivative


## 1 Visualizing functions that grow fast

Many processes in biology, like bacterial growth, are modelled with exponential functions. Exponential functions however grow incredibly quickly and as such it can be very difficult to visualize what is happening when x gets large.

Consider for example that we have 2 different strains of bacteria. One of which starts at an initial population described by the following functions.
$\mathrm{E} 1(\mathrm{x})=100+\exp (\mathrm{x})$
$\mathrm{E} 2(\mathrm{x})=\exp (2 \mathrm{x})$
Which of these populations starts at a higher value? Which population grows faster? Is there a value for x where both populations will have the same value?

First let us graph both of these functions normally on the same plot. To do this we are going to use the lines command in R .

```
> x<-seq(0,5,.01)
> E1<-100*exp(x)
> E2<-exp (2*x)
> plot(x,E1,xlab='days',ylab='population',main='Bacterial Growth')
> lines(x,E2)
```

We have now plotted both graphs of our bacterial population growth on the same plot. It is not very easy to see where they intersect though. In order to better visualize what is happening let us scale the population with a logarithm. This is called making a log plot.

```
> E1log<-log(E1)
> E2log<-log(E2)
```

now let us replot our graphs with this log scaling:

```
> plot(x,E1log,xlab='days',ylab='log population',main='log Bacterial Growth')
> lines(x,E2log)
```

What happened to the shapes of the population graphs, what types of functions do they look like now? Approximately where do the population graphs intersect? What population does the correspond to now?

## 2 Secant Lines, Tangents, and Approximating the Derivative

Let us consider a problem where we throw a ball into the air from the ground. We throw the ball straight up at a velocity of $20 \mathrm{~m} / \mathrm{s}$. The height of the ball as a function of time can be modelled as $Q(t)=-9.8 t^{2}+20 t$.

We would like to visualize this process as a graph. To do that let us introduce a new function in R , functions.

```
> f<-function(t){-9.8*t^2+20*t}
> t<-seq(0,2.5,.01)
> plot(t,f(t),type='l')
> lines(t,0*t)
```

This created a plot of the height of the ball as a function of time, where the lines $\left(\mathrm{t}, 0^{*} \mathrm{t}\right)$ created an image of the ground for us. What I would like to know is an approximation of the velocity of the ball when it has travelled for 1 second. To do this we can create secant lines going through the point $(1, \mathrm{f}(1))$ and another point on the graph. As the second point gets closer and closer to our point of interest $(1, f(1))$ we will be a better and better approximation.

To start off, let us get the x and y values of our secant points

```
> x1<-1
>y1<-f(x1)
> x2<-0.5
> y2<-f(x2)
> slope<-(y2-y1)/(x2-x1)
```

To get the equation of our secant line let us use point slope form $(y-y 1)=\operatorname{slope}^{*}(x-x 1)$

```
> y<-function(x){slope*(x-x1)+y1}
> lines(t,y(t))
```

Lets see what happens if we pick a point a little closer:

```
x x<-0.75
> y2<-f(x2)
> slope<-(y2-y1)/(x2-x1)
> y<-function(x){slope*(x-x1)+y1}
> lines(t,y(t))
```

Notice that the closer a point that we pick to $\mathrm{x}=1$, the better an approximation to the tangent line that we get. Finally let us pick a point really close and zoom in on what is happening.

```
> x2<-0.99
y2<-f(x2)
slope<-(y2-y1)/(x2-x1)
y<-function(x){slope(x-x1)+y1}
t<-seq(.97,1.02,.001)
plot(t,f(t),type='l')
> lines(t,y(t))
```

The tangent line has slope $-2^{*}(9.8)(1)+20=0.4$
And the equation of the tangent line is $y=0.4^{*}(x-1)+y 1$
Let us add this line to our graph.

```
slope<-0.4
y<-function(x){slope*(x-x1)+y1}
lines(t,y(t))
```

As you can see the secant line gave us a pretty good approximation.

## 3 Assignment for this week

1. Approximate the intersections of the functions $f 1(x)=x^{6}-100$ and $f 2(x)=\exp (x)$ How many intersections should we expect? Be sure to include a graph of your plot including both functions. (Hint: you may want to try a $\log$ scale to better visualize the problem, consider starting with a large domain as well so that you don't miss any)
2. For the ball problem from section 2: The ball reaches its maximum height after (20/19.6) seconds. Show a progression of secant lines to approximate the velocity of the ball when it is at its maximum height. Include in your answer the graph of the function with of all of your secant lines (at least 2)

Finally be sure to include your code with your final solutions.
You can easily do this with the command

```
> save.image("myfile.RData")
> savehistory("myfile.Rhistory")
```

