1170:Lab14

December 10th, 2013

Goals For This Week

In this fourteenth lab we will further explore autonomous differential equations, and introduce systems of autonomous differential equations.

1 **Autonomous Differential Equations**

Suppose we have a differential equation:

$$\frac{df}{dt} = g(f) \tag{1}$$

$$f(a) = f_a \tag{2}$$

$$(a) = f_a \tag{2}$$

Notice that the right hand side of the equation depends on the function f, not on the variable t. We say that such an equation is a (time) autonomous differential equation.

Consider as a new example a population of rabbits. The rabbits reproduce at a rate proportional to how many rabbits there are, and the die at a rate proportional to how many rabbits there are. Let us call the birth rate α , and the death rate β . Then the population of rabbits, R(t) will evolve according to the following differential equation.

$$\frac{dR}{dt} = \alpha R - \beta R = (\alpha - \beta)R \tag{3}$$

What will happen to population of rabbits, R, as time approaches infinity in each case $(\alpha > \beta, \beta > \alpha)$?

$\mathbf{2}$ **Predator-Prey Model**

Now suppose that we introduce a predator into the system. Wolves will eat the rabbits, which will increase the wolf population and decrease the rabbit population. Let's see how this would change the differential equations.

$$\frac{dR}{dt} = \alpha R - \beta R - \gamma R W \tag{4}$$

$$\frac{dW}{dt} = \delta RW - \epsilon W \tag{5}$$

Where: α is the growth rate of rabbits (assuming inexhaustible food supply) β is the natural death rate of rabbits γ is the predation rate constant (the rate at which wolves eat rabbits) δ is the rate at which wolves are born ϵ is the death rate for wolves

What can we say about the equilibrium of this system?

Suppose that $\alpha = 4, \beta = 1, \gamma = 1, \delta = 0.5$, and $\epsilon = 1$. Let us generate solution curves for this system using Euler's method. Suppose that we start with 100 rabbits and 2 wolves.

```
alpha<-4
beta<-1
gamma<-1
delta<-0.5
epsilon<-1
dR<-function(R,W){alpha*R-beta*R-gamma*R*W}
dW<-function(R,W){delta*R*W-epsilon*W}
a<-0
b<-50
n<-1000
t<-seq(1,n+1)
R < -seq(1, n+1)
W<-seq(1,n+1)
t[1]<-a
R[1]<-100
₩[1]<-2
delt<-(b-a)/n
for (i in 2:(n+1)) {t[i]<-t[i-1]+delt
R[i]<-dR(R[i-1],W[i-1])*delt+R[i-1]
W[i]<-dW(R[i-1],W[i-1])*delt+W[i-1]}
plot(t,R,type='1')
lines(t,W)
```

Notice that our differential equations for the populations of rabbits and wolves were functions of both the number of rabbits and the number of wolves. Similarly we need to track both populations in our Euler's Method code.

What happens if we change b, our final time of the system?

What about if we change our other parameters? What is happening to the populations?

3 Assignment for this week

For the assignment this week consider the SIR model, which tracks the number of people in a population that are susceptible to a disease, infected with the disease, or recovered from the disease. The model is as follows:

$$\frac{dS}{dt} = -\alpha SI\tag{6}$$

$$\frac{dI}{dt} = \alpha SI - \beta I \tag{7}$$

$$\frac{dR}{dt} = \beta I \tag{8}$$

Suppose that $\alpha = 2, \beta = 1$, that the initial number of susceptible people is 10000, that there is 1 infected person initially, and that no one is recovered initially. Be sure to run your code for long enough (b is large enough) that the solutions reach equilibrium.

- 1. Write an Euler's code method which will generate a solution curve for this system. Now change the parameters α and β but keep $\alpha > \beta$ What changed?
- 2. Now assume that $\beta > \alpha$ and repeat the steps from problem 1 with $\beta = 2$, $\alpha = 1$. What happens to the populations? Explain why you think that is.
- 3. Submit your codes and a couple of plots of the solution curves.