1170:Lab13

December 3rd, 2013

Goals For This Week

In this thirteenth lab we will explore autonomous differential equations, and interesting bifurcations which can arise as parameters change.

1 Autonomous Differential Equations

Suppose we have a differential equation:

$$\frac{df}{dt} = g(f) \tag{1}$$

$$f(a) = f_a \tag{2}$$

Notice that the right hand side of the equation depends on the function f, not on the variable t. We say that such an equation is a (time) autonomous differential equation.

2 Bifurcations

As parameter values in an equation change sometimes the number and stability of the equilibria will change as well. Such an occurrence is called a bifurcation, and their study is a central topic of differential equations.

Consider the equation:

$$\frac{dx}{dt} = cx - x^2 \tag{3}$$

We wish to study the dynamics of solutions to this differential equation as the parameter c changes. Notice that this is not the same thing as c being a function of t.

First we wish to find the equilibria as a function of c. In other words:

$$\frac{dx}{dt} = cx - x^2 = 0 \tag{4}$$

$$cx - x^2 = 0 \tag{5}$$

$$x(c - x) = 0 \tag{6}$$

Thus there are equilibria at x=0, and x=c. As the value of the parameter c changes from -1 to 1, the equilibrium at x=c will move closer and closer to the equilibrium at x=0, then it will disappear and there will just be the 1 equilibrium at x=0 when c=0, and finally it will reappear again and move further and further away. This type of bifurcation at c=0 is called a transcritical bifurcation. What happens to the stability of the equilibria as c changes?

3 Exploring Bifurcations Numerically

Now lets plot solution curves for this equation with initial condition x(0)=0, x(0)=-2, x(0)=2 using Euler's method for 5 values of c changing from c=-1 to c=1. (c=-1, c=-0.5, c=0, c=0.5, c=1)

```
c=-1
g<-function(x){c*x-x^2}
a<-0
b<-5
n<-1000
t<-seq(1,n+1)
f<-seq(1,n+1)
t[1]<-a
f[1]<- 2
delt<-(b-a)/n
for (i in 2:(n+1)) {t[i]<-t[i-1]+delt
f[i]<-g(f[i-1])*delt+f[i-1]}
plot(t,f,type='l')
```

What is happening, let us compare that to our bifurcation diagrams and our phase planes for this equation.

4 Assignment for this week

Consider this time the autonomous differential equation:

$$\frac{dx}{dt} = c - x^2 \tag{7}$$

Analyze what happens to the equilibria as c changes from -1 to 1. Does the stability of the equilibria change?

1. Submit your picture of the bifurcation diagram, like we did in class, for this differential equation as c changes from -1 to 1.
2. Given initial conditions x(0)=-2, x(0)=0, and x(0)=2 discuss what the solution curves will look like if c=-1, c=0, or c=1. Note: there are 9 cases to consider.
3. Submit a couple of the more interesting plots of the solution curves using Euler's method in R.