

Functions

A *function* is a way of describing a relationship between two sets.

To have a function we first need two sets, so let's suppose that D and T are sets. Then a *function* is something that assigns every $x \in D$ to a single object in T .

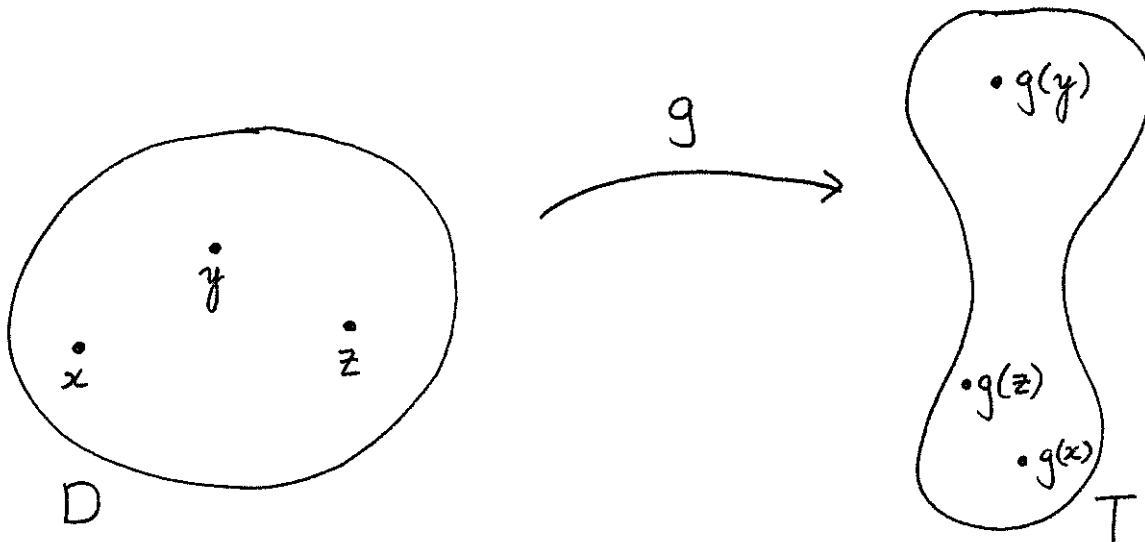
D is called the *domain* of the function, and T is called the *target* of the function. We usually assign names to our functions — though usually simple and generic names — like g , for example. Naming the function lets us give a specific name to the object in the target that the function assigns to an object in the domain as follows:

If $x \in D$, then $g(x) \in T$ is the object that g assigns to x .

Writing the symbols

$$g : D \rightarrow T$$

is a shorthand for writing that g is a function that assigns every $x \in D$ to a single object in T .



Example 1. $f : \mathbb{N} \rightarrow \mathbb{R}$ where $f(n) = 2^n$

- $f(3) = 2^3 = 2 \cdot 2 \cdot 2 = 8$
- $f(4) = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$
- $f(1) = 2^1 = 2$

Example 2. $g : \mathbb{R} \rightarrow \mathbb{R}$ where $g(x) = 3x - 4$

- $g(2) = 3 \cdot 2 - 4 = 2$
- $g(-1) = 3 \cdot (-1) - 4 = -3 - 4 = -7$

Example 3. *Constant functions* are functions that assign every object in the domain to the same object in the target. For example, $h : \mathbb{R} \rightarrow \mathbb{R}$ where $h(x) = \frac{-5}{3}$.

Example 4. The *identity function* is the function that assigns every object in the domain to itself. (To have an identity function, the domain and target have to be the same set.) Identity functions are important enough that they get to have a name that is reserved only for identity functions: *id*. In other words, the identity function is described by

$$id : \mathbb{R} \rightarrow \mathbb{R} \text{ where } id(x) = x.$$

(Sometimes it will make sense for us to use a different domain for the function *id*, but that's mostly a cosmetic change; it doesn't affect the way the function acts.)

Not a function I.

Assign to every $n \in \mathbb{N}$ the number $x \in \mathbb{R}$ such that $x^2 = n$.

Not a function II.

Assign to every $x \in \mathbb{R}$ the real number $\frac{3}{x-2}$.

Question: Why aren't the two examples above functions?

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Exercises

Suppose $f : \mathbb{N} \rightarrow \mathbb{R}$ is defined by $f(n) = \frac{1}{n^2}$.

1) What is $f(5)$?

2) What is $f(10)$?

Suppose $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = \frac{2x^2 - x + 1}{x^2 + 1}$.

3) What is $g(1)$?

4) What is $g(-3)$?

Suppose $h : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $h(x) = 14$.

5) What is $h(0)$?

6) What is $h(\frac{\pi^2}{6})$?

7) What is $id(15)$?

8) What is $id(-4)$?

You have a stubborn puppy who won't come when she's called unless you give her three puppy treats.

9) How many puppy treats do you need to bring with you on a walk if you expect to call the puppy seven times?

10) How many puppy treats do you need to bring with you on a walk if you expect to call the puppy ten times?

What's the domain and target of this "puppy function"?