Substitution

In this chapter, we'll examine systems of two linear equations in two variables that have unique solutions. If a system has a unique solution, we can use a method called "substitution" to find the unique solution.

How to find the solution

Suppose you're given a system of two linear equations in two variables, and that the variables are named x and y. Name the equations "Equation-1" and "Equation-2" (the order doesn't matter).

Use algebra to transform Equation-1 into an equation that looks like

$$x = (\text{something with } y\text{'s and numbers})$$

Let's call this equation "New-equation-1".

Use New-equation-1 to substitute for x in Equation-2. You'll be left with a "New-equation-2" that only has y's and numbers — there won't be any x's.

Use New-equation-2 to solve for y. Once you have, substitute your solution for y into New-equation-1. That will tell you what x is.

(In the explanation above, the roles of x and y could have been switched.)

Problem 1. Find the solution to the system

$$\begin{aligned} x + 4y &= -2 \\ 2x + 7y &= -3 \end{aligned}$$

Solution. Let's name $x + 4y = -2$ Equation-1, and solve for x. Then we'll get that

$$x = -4y - 2$$

This is New-equation-1.

Equation-2 is $2x + 7y = -3$. Using New-equation-1, we can replace x in Equation-2 with $-4y - 2$ to get

$$2[-4y - 2] + 7y = -3$$

This is New-equation-2, and we can use it to solve for y:

$$-8y - 4 + 7y = -3$$

thus

$$-y = 1$$

and hence

$$y = -1$$

Now that we know y, we return to New-equation-1, replace y with -1, and we are left with

$$x = -4(-1) - 2 = 2$$

Now we know that $x = 2$ and $y = -1$ is the solution to the system of equations that we started with.

Problem 2. Find the solution to the system

$$-2x + y = -1$$
$$5x - 2y = 5$$

Solution. Use the first equation to solve for x:

$$x = \frac{y+1}{2}$$

Substitute for x in the second equation:

$$5\left[\frac{y+1}{2}\right] - 2y = 5$$

so

$$\frac{5y}{2} + \frac{5}{2} - 2y = 5$$

and then

$$\frac{y}{2} + \frac{5}{2} = 5$$

Multiplying both sides of the equation by 2 gives

$$y + 5 = 10$$

and therefore,

$$y = 5$$

Now return to the equation

$$x = \frac{y+1}{2}$$

and substitute 5 for y to get

$$x = \frac{5+1}{2}$$

which means that

$$x = 3$$

We have our solution to the system, it's $x = 3$ and $y = 5$.

*   *   *   *   *   *   *   *   *   *   *   *   *

Exercises

Each of the systems below has a unique solution. Find the solution.

1.)

$$\begin{aligned} 8x + 4y &= 12 \\ x - 7y &= -21 \end{aligned}$$

2.)

$$\begin{aligned} 10x - 3y &= 52 \\ -3x + y &= -16 \end{aligned}$$

3.)

$$\begin{aligned} 3x &= 5 \\ 2x - 3y &= 12 \end{aligned}$$

4.)

$$\begin{aligned} 2x + 8y &= -8 \\ -3x + 6y &= 12 \end{aligned}$$