

Math 1050-006 Midterm 1 Practice Test Solutions

1.) Is $\{3, \frac{5}{7}, \frac{-3}{-65}, -271\} \subseteq \mathbb{Q}$? Why or why not?

Answer: True, each of the numbers in the subset is a rational number

so $\{3, \frac{5}{7}, \frac{-3}{-65}, -271\}$ is a subset of \mathbb{Q}

2.) What is $\{1,7,8\} - \{1,8,9\}$?

Answer: As 1,8 are objects shared by both sets $\{1,7,8\} - \{1,8,9\} = \{1,7,8\} - \{1,8\} = \{7\}$

3.) Is $[0, \infty) \subseteq \mathbb{R} - \{\pi\}$ a true statement? Why or why not?

Answer: False, $\pi \in [0, \infty)$ but $\pi \notin \mathbb{R} - \{\pi\}$

4.) Is $[-17, \infty) \subseteq (-17, \infty)$ a true statement? Why or why not?

Answer: False, $-17 \in [-17, \infty)$ but $-17 \notin (-17, \infty)$

5.) Find 3 things that are wrong with the following statement:

$$[2, -1] \in [-\infty, \infty)$$

Answer: 1.) $[2, -1]$ should be $[-1, 2]$ as $2 > -1$

2.) wrong contained in symbol, \in should be \subseteq

3.) Wrong notation, $[-\infty, \infty)$ should be $(-\infty, \infty)$

6.) Suppose $f: \mathbb{N} \rightarrow \mathbb{R}$ is defined by $f(n) = \frac{1}{n^2}$

(a) n is an object of which set? (Rational numbers, Integers, Real numbers, or Natural numbers)?

Answer: n is an object of the Natural numbers since the Domain = \mathbb{N} .

(b) What is $f(2)$?

$$\text{Answer: } f(2) = \frac{1}{(2)^2} = \frac{1}{4}$$

(c) What is $f(-3)$?

Answer: $f(-3)$ is not defined as $-3 \notin \mathbb{N}$

7.) Suppose $h: \mathbb{R} \rightarrow \mathbb{R}$ is an identity function ($h(x)=id(x)$)

(a) What is $h(\pi)$?

Answer: $h(\pi)=\pi$

(b) What is $h(0)$?

Answer: $h(0)=0$

8.) (a) What is the formula for an arithmetic sequence?

Answer: $a_{n+1}=a_n+d$

(b) If $a_1, a_2, a_3, \dots = 3, -1, -5, \dots$

what do a_1 and d equal for this sequence? (i.e. fill in the unknowns in the arithmetic sequence formula from (a))

Answer: $a_1=3, d=-4$ since $a_2=a_1+d \Rightarrow -1=3+d \Rightarrow d=-4$

(c) What is the prediction equation for an arithmetic sequence?

Answer: $a_n=a_1+(n-1)d$

(d) Use your solution in part (c) to predict the 500th term of the sequence given in part (b)

Answer: $a_{500}=a_1+(500-1)(-4)=3+(499)(-4)=(3-1996)=-1993$

9.) What is the 9th term in the sequence

$$25, 15, 9, \frac{27}{5}, \dots$$

Answer: This sequence is a geometric sequence, therefore we use the geometric sequence prediction equation

$$a_n = r^{(n-1)} a_1 \text{ and for this sequence } a_1=25 \text{ and } r=\frac{3}{5} \text{ therefore } a_9 = \left(\frac{3}{5}\right)^8 (25) = \frac{3^8}{5^6}$$

10.) (a) What is the equation for the sum of a geometric series, and what condition do we have on (r) in order to use this equation? (recall: $a_{n+1} = r \cdot a_n$)

Answer: $\frac{a_1}{(1-r)}$ where $-1 < r < 1$

(b) Using your solution in part (a) what is the sum of all of the terms in the sequence

$$2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \dots$$

Answer: This sequence is a geometric sequence with $a_1=2$, $r = \frac{1}{4}$

Therefore using our equation from part (a) we get that the sum is:

$$\frac{2}{(1-\frac{1}{4})} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$$

11.) What is the sum of the first 30 terms of the sequence: 5, 3, 1, -1, ...

Answer: This is an arithmetic sequence with $a_1=5$ and $d=-2$

We can therefore use the finite sum of an arithmetic sequence equation to get:

$$\text{Sum} = \frac{n}{2}(a_1 + a_n) \text{ where } n=30$$

$$\text{Therefore } \sum_{i=1}^{30} a_i = \frac{30}{2}(5 + a_n) \text{ where } a_n = a_1 + (30-1)(-2) = 5 + (29)(-2) = -53$$

$$\text{Therefore } \sum_{i=1}^{30} a_i = \frac{30}{2}(5 - 53) = (15)(-48)$$

12.) What is $\sum_{k=1}^{50} 2k$

Answer: $\sum_{k=1}^{50} 2k = 2 \sum_{k=1}^{50} k$ which is 2 times the sum of the first 50 terms of the arithmetic sequence

with $a_1=1$, $d=1$ which we can solve using the sum equation to get:

$$2 \sum_{k=1}^{50} k = 2 \frac{50}{2}(1+50) = (50)(51) = 2550$$

15.) (b) Now use the Binomial Theorem and Pascal's triangle to solve $(x+y)^5$

Answer: $\sum_{i=0}^n \binom{n}{i} (x)^i (y)^{(n-i)}$

$$\sum_{i=0}^5 \binom{5}{i} (x)^i (y)^{(5-i)} = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

(c) Using your result from part (b) solve $(z-3)^5$

Answer: Let $x=z$ and $y=-3$ and plug in to the solution

$$z^5 + 5z^4(-3) + 10z^3(-3)^2 + 10z^2(-3)^3 + 5z(-3)^4 + (-3)^5 = z^5 - 15z^4 + 90z^3 - 270z^2 + 405z - 243$$

16.) What is the implied domain of $h(x) = \frac{4x-5}{x-2}$

Answer: if $x=2$ we divide by zero, and we only divide by zero if $x=2$, therefore the implied domain is $\mathbb{R} - \{2\}$

17.) If $f(x) = x^2 - 3$, $g(x) = x+5$ solve:

(a) $f \circ g(x)$

Answer: $f(g(x)) = f(x+5) = (x+5)^2 - 3 = x^2 + 10x + 22$

(b) $g \circ f(x)$

Answer: $g(f(x)) = g(x^2 - 3) = x^2 - 3 + 5 = x^2 + 2$