## Math 1050-006 Midterm 1 Practice Test Solutions

1.) Is $\left\{3, \frac{5}{7}, \frac{-3}{-65},-271\right\} \subseteq \mathbb{Q}$ ? Why or why not?

Answer: True, each of the numbers in the subset is a rational number so $\left\{3, \frac{5}{7}, \frac{-3}{-65},-271\right\}$ is a subset of $\mathbb{Q}$
2.) What is $\{1,7,8\}-\{1,8,9\}$ ?

Answer: As 1,8 are objects shared by both sets $\{1,7,8\}-\{1,8,9\}=\{1,7,8\}-\{1,8\}=\{7\}$
3.) Is ) $[0, \infty) \subseteq \mathbb{R}-\{\pi\}$ a true statement? Why or why not?

Answer: False, $\pi \in[0, \infty)$ but $\pi \notin \mathbb{R}-\{\pi\}$
4.) Is $[-17, \infty) \subseteq(-17, \infty)$ a true statement? Why or why not?

Answer: False, $-17 \in[-17, \infty]$ but $-17 \notin(-17, \infty)$
5.) Find 3 things that are wrong with the following statement:

$$
[2,-1] \in[-\infty, \infty)
$$

Answer: 1.) $[2,-1]$ should be $[-1,2]$ as $2>-1$
2.) wrong contained in symbol, $\in$ should be $\subseteq$
3.) Wrong notation, $[-\infty, \infty)$ should be $(-\infty, \infty)$
6.) Suppose $f: \mathbb{N} \rightarrow \mathbb{R}$ is defined by $f(n)=\frac{1}{n^{2}}$
(a) n is an object of which set? (Rational numbers, Intergers, Real numbers, or Natural numbers)?

Answer: n is an object of the Natural numbers since the Domain $=\mathbb{N}$.
(b) What is $f(2)$ ?

Answer: $f(2)=\frac{1}{(2)^{2}}=\frac{1}{4}$
(c) What is $\mathrm{f}(-3)$

Answer: $\mathrm{f}(-3)$ is not defined as $-3 \notin \mathbb{N}$
7.) Suppose $h: \mathbb{R} \rightarrow \mathbb{R}$ is an identity function $(h(x)=i d(x))$
(a) What is $h(\pi)$ ?

Answer: $\mathrm{h}(\pi)=\pi$
(b) What is $\mathrm{h}(0)$ ?

Answer: $\mathrm{h}(0)=0$
8.) (a) What is the formula for an aritmetic sequence?

Answer: $\mathrm{a}_{\mathrm{n}+1}=\mathrm{a}_{\mathrm{n}}+\mathrm{d}$
(b) If $a_{1}, a_{2}, a_{3}, \ldots=3,-1,-5, \ldots$
what do $a_{1}$ and $d$ equal for this sequence? (i.e. fill in the unknowns in the arithmetic sequence formula from (a))

Answer: $a_{1}=3, d=-4$ since $a_{2}=a_{1}+d=>-1=3+d=>d=-4$
(c) What is the prediction equation for an arithmetic sequence?

Answer: $\mathrm{a}_{n}=\mathrm{a}_{1}+(\mathrm{n}-1) \mathrm{d}$
(d) Use your solution in part (c) to predict the $500^{\text {th }}$ term of the seqence given in part (b)

Answer: $\mathrm{a}_{500}=\mathrm{a}_{1}+(500-1)(-4)=3+(499)(-4)=(3-1996)=-1993$
9.) What is the $9^{\text {th }}$ term in the sequence

$$
25,15,9, \frac{27}{5}, \ldots
$$

Answer: This sequence is a geometric sequence, therefore we use the geometric sequence prediction equation $a_{n}=r^{(n-1)} a_{1}$ and for this sequence $a_{1}=25$ and $r=\frac{3}{5}$ therefore $a_{9}=\left(\frac{3}{5}\right)^{8}(25)=\frac{3^{8}}{5^{6 .}}$
10.) (a) What is the equation for the sum of a geometric series, and what condition do we have on (r) in order to use this equation? (recall: $a_{n+1}=r \cdot a_{n}$ )

Answer: $\frac{\mathrm{a}_{1}}{(1-\mathrm{r})}$ where $-1<\mathrm{r}<1$
(b) Using your solution in part (a) what is the sum of all of the terms in the sequence

$$
2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \ldots .
$$

Answer: This sequence is a geometric sequence with $\mathrm{a}_{1}=2, r=\frac{1}{4}$
Therefore using our equation from part (a) we get that the sum is:
$\frac{2}{\left(1-\frac{1}{4}\right)}=\frac{2}{\frac{3}{4}}=\frac{8}{3}$
11.) What is the sum of the first 30 terms of the sequence: $5,3,1,-1, \ldots$

Answer: This is an arithmetic sequence with $\mathrm{a}_{1}=5$ and $\mathrm{d}=-2$
We can therefore use the finite sum of an arithmetic sequence equation to get:
Sum $=\frac{\mathrm{n}}{2}\left(a_{1}+a_{\mathrm{n}}\right)$ where $\mathrm{n}=30$
Therefore $\sum_{i=1}^{30} a_{i}=\frac{30}{2}\left(5+a_{n}\right)$ where $\mathrm{a}_{n}=\mathrm{a}_{1}+(30-1)(-2)=5+(29)(-2)=-53$
Therefore $\sum_{i=1}^{30} a_{i}=\frac{30}{2}(5-53)=(15)(-48)$
12.) What is $\sum_{k=1}^{50} 2 \mathrm{k}$

Answer: $\sum_{k=1}^{50} 2 \mathrm{k}=2 \sum_{k=1}^{50} \mathrm{k}$ which is 2 times the sum of the first 50 terms of the arithmetic sequence with $\mathrm{a}_{1}=1, \mathrm{~d}=1$ which we can solve using the sum equation to get:
$2 \sum_{k=1}^{50} \mathrm{k}=2 \frac{50}{2}(1+50)=(50)(51)=2550$
13.) A combination lock has forty numbers to pick from (1-40) and a combintion is a list of 3 of these numbers where no two adjacent numbers are the same.
(i.e $(38,7,4)$ is a combination, but $(38,38,4)$ is not)

How many different combinations can a combination lock have?
Answer: There are 40 choices you can make for the first number in the combination, 39 choices for the second number, and 39 choices for the third number This results in (40)(39) ${ }^{2}$ possible combinations
14.) To play the North Carolina pick 5 lottery you need to create a ticket by picking any 5 of the 60 number choices $(1-60)$ where the order of the choices does not matter. How many different lottery tickets are there?

Answer: $\binom{60}{5}=\frac{60!}{(60-5)!(5)!}=\frac{60!}{(55)!(5)!}=\frac{(60)(59)(58)(57)(56)}{(5)(4)(3)(2)(1)}$
15.) (a) Write out the first 6 rows of Pascal's triangle (row 0 up to row 5)

Answer:

15.) (b) Now use the Binomial Theorem and Pascal's triangle to solve $(x+y)^{5}$

Answer: $\sum_{i=0}^{n}\binom{n}{i}(x)^{i}(y)^{(n-i)}$
$\sum_{i=0}^{5}\binom{5}{i}(x)^{i}(y)^{(5-i)}=x^{5}+5 \mathrm{x}^{4} y+10 \mathrm{x}^{3} y^{2}+10 \mathrm{x}^{2} y^{3}+5 \mathrm{xy}^{4}+y^{5}$
(c) Using your result from part (b) solve (z-3) ${ }^{5}$

Answer: Let $\mathrm{x}=\mathrm{z}$ and $\mathrm{y}=-3$ and plug in to the solution
$z^{5}+5 z^{4}(-3)+10 z^{3}(-3)^{2}+10 z^{2}(-3)^{3}+5 z(-3)^{4}+(-3)^{5}=z^{5}-15 z^{4}+90 z^{3}-270 z^{2}+405 z-243$
16.) What is the implied domain of $h(x)=\frac{4 x-5}{x-2}$

Answer: if $\mathrm{x}=2$ we divide by zero, and we only divide by zero if $\mathrm{x}=2$, therefore the implied domain is $\mathbb{R}-\{2\}$
17.) If $f(x)=x^{2}-3, g(x)=x+5$ solve:
(a) $f \circ g(x)$

Answer: $\mathrm{f}(\mathrm{g}(\mathrm{x}))=\mathrm{f}(\mathrm{x}+5)=(\mathrm{x}+5)^{2}-3=x^{2}+10 \mathrm{x}+22$
(b) $g \circ f(x)$

Answer: $\mathrm{g}(\mathrm{f}(\mathrm{x}))=\mathrm{g}\left(x^{2}-3\right)=\mathrm{x}^{2}-3+5=\mathrm{x}^{2}+2$

