Math 1050-006 Midterm 1 Practice Test Solutions

1.) Is $\{3, \frac{5}{7}, \frac{-3}{-65}, -271\} \subseteq \mathbb{Q}$? Why or why not?

Answer: True, each of the numbers in the subset is a rational number

so
$$\{3, \frac{5}{7}, \frac{-3}{-65}, -271\}$$
 is a subset of **Q**

2.) What is {1,7,8}-{1,8,9}?

Answer: As 1,8 are objects shared by both sets {1,7,8}-{1,8,9}={1,7,8}-{1,8}={7}

3.) Is $(0,\infty) \subseteq \mathbb{R} - \{\pi\}$ a true statement? Why or why not?

Answer: False, $\pi \in [0, \infty)$ but $\pi \notin \mathbb{R} - \{\pi\}$

4.) Is $[-17, \infty) \subseteq (-17, \infty)$ a true statement? Why or why not?

Answer: False, $-17 \in [-17, \infty]$ but $-17 \notin (-17, \infty)$

- 5.) Find 3 things that are wrong with the following statement: $[2,-1] \in [-\infty,\infty)$
- Answer: 1.) [2,-1] should be [-1,2] as 2 > -12.) wrong contained in symbol, \in should be \subseteq 3.) Wrong notation, $[-\infty, \infty)$ should be $(-\infty, \infty)$
- 6.) Suppose f: $\mathbb{N} \rightarrow \mathbb{R}$ is defined by f(n)= $\frac{1}{n^2}$

(a) n is an object of which set ? (Rational numbers, Intergers, Real numbers, or Natural numbers)?

Answer: n is an object of the Natural numbers since the Domain= \mathbb{N} .

(b) What is f(2)?

Answer: $f(2) = \frac{1}{(2)^2} = \frac{1}{4}$

(c) What is f(-3)

Answer: f(-3) is not defined as $-3 \notin \mathbb{N}$

- 7.) Suppose h: R→R is an identity function (h(x)=id(x))
 (a) What is h(π)?
- Answer: $h(\pi) = \pi$
 - (b) What is h(0)?

Answer: h(0)=0

8.) (a) What is the formula for an aritmetic sequence?

Answer: $a_{n+1} = a_n + d$

(b) If a₁, a₂, a₃, ... = 3, -1, -5,...
what do a₁ and d equal for this sequence? (i.e. fill in the unknowns in the arithmetic sequence formula from (a))

Answer: $a_1 = 3$, d = -4 since $a_2 = a_1 + d = -1 = 3 + d = -4$

(c) What is the prediction equation for an arithmetic sequence?

Answer: $a_n = a_1 + (n-1)d$

(d) Use your solution in part (c) to predict the 500th term of the seqence given in part (b)

Answer: $a_{500} = a_1 + (500-1)(-4) = 3 + (499)(-4) = (3-1996) = -1993$

9.) What is the 9th term in the sequence

25, 15, 9,
$$\frac{27}{5}$$
, ...

Answer: This sequence is a geometric sequence, therefore we use the geometric sequence prediction equation

$$a_n = r^{(n-1)}a_1$$
 and for this sequence $a_1 = 25$ and $r = \frac{3}{5}$ therefore $a_9 = (\frac{3}{5})^8 (25) = \frac{3^8}{5^{6}}$

10.) (a) What is the equation for the sum of a geometric series, and what condition do we have on (r) in order to use this equation? (recall: $a_{n+1} = r \cdot a_n$)

Answer:
$$\frac{a_1}{(1-r)}$$
 where -1 < r <1

(b) Using your solution in part (a) what is the sum of all of the terms in the sequence

2,
$$\frac{1}{2}$$
, $\frac{1}{8}$, $\frac{1}{32}$,...

Answer: This sequence is a geometric sequence with $a_1 = 2$, $r = \frac{1}{4}$

Therefore using our equation from part (a) we get that the sum is:

$$\frac{2}{(1-\frac{1}{4})} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$$

11.) What is the sum of the first 30 terms of the sequence: 5, 3, 1, -1, ...

Answer: This is an arithmetic sequence with $a_1 = 5$ and d = -2

We can therefore use the finite sum of an arithmetic sequence equation to get:

Sum=
$$\frac{n}{2}(a_1+a_n)$$
 where n=30
Therefore $\sum_{i=1}^{30} a_i = \frac{30}{2}(5+a_n)$ where $a_n = a_1 + (30-1)(-2) = 5 + (29)(-2) = -53$
Therefore $\sum_{i=1}^{30} a_i = \frac{30}{2}(5-53) = (15)(-48)$

12.) What is $\sum_{k=1}^{50} 2k$

Answer: $\sum_{k=1}^{50} 2k = 2 \sum_{k=1}^{50} k$ which is 2 times the sum of the first 50 terms of the arithmetic sequence with $a_1 = 1$, d = 1 which we can solve using the sum equation to get:

$$2\sum_{k=1}^{50} k = 2\frac{50}{2}(1+50) = (50)(51) = 2550$$

13.) A combination lock has forty numbers to pick from (1 - 40) and a combinition is a list of 3 of these numbers where no two adjacent numbers are the same.(i.e (38, 7, 4) is a combination, but (38, 38, 4) is not)

How many different combinations can a combination lock have?

Answer: There are 40 choices you can make for the first number in the combination, 39 choices for the second number, and 39 choices for the third number This results in $(40)(39)^2$ possible combinations

14.) To play the North Carolina pick 5 lottery you need to create a ticket by picking any 5 of the 60 number choices (1 - 60) where the order of the choices does not matter. How many different lottery tickets are there?

Answer:
$$\binom{60}{5} = \frac{60!}{(60-5)!(5)!} = \frac{60!}{(55)!(5)!} = \frac{(60)(59)(58)(57)(56)}{(5)(4)(3)(2)(1)}$$

15.) (a) Write out the first 6 rows of Pascal's triangle (row 0 up to row 5)

Answer:

15.) (b) Now use the Binomial Theorem and Pascal's triangle to solve $(x+y)^5$

Answer:
$$\sum_{i=0}^{n} {n \choose i} (x)^{i} (y)^{(n-i)}$$
$$\sum_{i=0}^{5} {5 \choose i} (x)^{i} (y)^{(5-i)} = x^{5} + 5x^{4}y + 10x^{3}y^{2} + 10x^{2}y^{3} + 5xy^{4} + y^{5}$$

(c) Using your result from part (b) solve $(z-3)^5$

Answer: Let x=z and y=-3 and plug in to the solution $z^{5}+5z^{4}(-3)+10z^{3}(-3)^{2}+10z^{2}(-3)^{3}+5z(-3)^{4}+(-3)^{5} = z^{5}-15z^{4}+90z^{3}-270z^{2}+405z-243$

16.) What is the implied domain of $h(x) = \frac{4x-5}{x-2}$

Answer: if x=2 we divide by zero, and we only divide by zero if x=2, therefore the implied domain is $\mathbb{R} - \{2\}$

17.) If $f(x) = x^2 - 3$, g(x) = x+5 solve:

(a)
$$f \circ g(x)$$

Answer: $f(g(x)) = f(x+5) = (x+5)^2 - 3 = x^2 + 10x + 22$

(b) $g \circ f(x)$

Answer: $g(f(x)) = g(x^2-3) = x^2-3+5 = x^2+2$