Homework 9 Solutions

Rows and Columns: 3, 5, 7

3) $\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ -2 \end{pmatrix} = (2)(8) + (3)(-2)$

$= 16 - 6 = \boxed{10}$

5) $\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = (1)(3) + (0)(4) + (1)(1)$

$= 3 + 0 + 1 = \boxed{4}$

7) $\begin{pmatrix} 4 & 6 & -3 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} = (4)(-2) + (6)(4) + (-3)(-2)$

$= -8 + 24 + 6 = \boxed{22}$

Vectors and Scalars: 2, 3, 4

2) $\begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} = \begin{pmatrix} 4+3 \\ 2+(-8) \\ 6+0 \end{pmatrix} = \begin{pmatrix} 7 \\ -6 \\ 6 \end{pmatrix}$

3) $2\begin{pmatrix} 7 \\ -3 \end{pmatrix} = \begin{pmatrix} (2)(7) \\ (2)(-3) \end{pmatrix} = \begin{pmatrix} 14 \\ -6 \end{pmatrix}$

4) $5\begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 5(-1) \\ 5(0) \\ 5(4) \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 20 \end{pmatrix}$

2×2 Matrices : 2, 3, 5

2) $$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2(3)+1(5) \\ 1(3)+1(5) \end{pmatrix} = \begin{pmatrix} 11 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2(-4)+1(6) \\ 1(-4)+1(6) \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

3) $$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 2(3)+1(5) & 2(-4)+1(6) \\ 1(3)+1(5) & 1(-4)+1(6) \end{pmatrix}$$

$$= \begin{pmatrix} 11 & -2 \\ 8 & 2 \end{pmatrix}$$

5) $$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$

Identity

$$IA = A$$

3×3 Matrices: 1, 3, 5

1) $$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & -3 \\ 3 & -2 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1(0)+1(1)+1(4) \\ 0(0)+2(1)+(-3)(4) \\ 3(0)+(-2)(1)+0(4) \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & -3 \\ 3 & -2 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1(-2)+1(1)+1(0) \\ 0(-2)+2(1)+(-3)(0) \\ 3(-2)+-2(1)+0(0) \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -8 \end{pmatrix}$$

3) $$\begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 & 5 \\ 0 & 3 & 1 \\ 0 & 0 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 0(2)+1(0)+0(0) & 0(-1)+1(3)+0(0) & 0(5)+1(1)+0(-4) \\ 2(2)+0(0)+2(0) & 2(-1)+0(3)+2(0) & 2(5)+0(1)+2(-4) \\ 0(2)+3(0)+0(0) & 0(-1)+3(3)+0(0) & 0(5)+3(1)+0(-4) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 3 & 1 \\ 4 & -2 & 2 \\ 0 & 9 & 3 \end{pmatrix}$$

5) $$\begin{pmatrix} 3 & -17 & 5 \\ 17 & 3 & 34 \\ 41 & 3 & 18 \end{pmatrix}\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Identity}} \Rightarrow \begin{pmatrix} 3 & -17 & 5 \\ 17 & 3 & 34 \\ 41 & 3 & 18 \end{pmatrix}$$

$$AI = A$$

Determinants and Inverse Matrices : 1-7, 10, 11

1) $\det \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = 2(1) - 1(1) = \boxed{1}$

2) $\det \begin{pmatrix} 1 & \pi \\ 0 & 1 \end{pmatrix} = 1(1) - \pi(0) = \boxed{1}$

3) $\det \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} = (-4)(-1) - (2)(2) = \boxed{0}$

4) $\det \begin{pmatrix} 3 & 0 & 0 \\ 107 & 1 & 0 \\ \sqrt{2} & 2 & 6 \end{pmatrix} = 3 \det \begin{pmatrix} 1 & 0 \\ 2 & 6 \end{pmatrix} = 3\left((1)(6) - 0(2)\right)$

$= \boxed{18}$

5) $\det \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix} = 1 \det \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} - 1 \det \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} + 1 \det \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$

$= \det \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} = 2(3) - 2(3) = \boxed{0}$

6) $\det \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 4 \end{pmatrix} = 1 \det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - 2 \det \begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix} + 1 \det \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$

$= \left(1(4) - 2(3)\right) - 2\left(2(4) - 2(1)\right) + \left(2(3) - 1(1)\right)$

$= (4 - 6) - 2(8 - 2) + (6 - 1)$

$= -2 - 12 + 5 = \boxed{-9}$

7) Which of the 6 matrices in 1-6 have an inverse?

1, 2, 4, 6 because they have non zero determinants

10) What is the inverse of $\begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}$?

determinant = $(4)(3) - (2)(-1) = 12 + 2 = 14$

$$\Rightarrow \begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}^{-1} = \frac{1}{14}\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$$

11) Are the following pairs of matrices inverses of each other?

$$\begin{pmatrix} 4 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 3 \\ -1 & 0 & 2 \end{pmatrix}$$

if you multiply them together you should get the identity matrix

$$= \begin{pmatrix} 4(0)+1(1)+2(-1) & 4(2)+1(1)+2(0) & 4(1)+1(3)+2(2) \\ & & \end{pmatrix}$$

already we can tell they aren't since so far we have

$$\begin{pmatrix} -1 & 9 & 11 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

Matrix Equations : 1, 2, 5

1)

$$\begin{aligned} 2x + 4y &= -7 \\ -x + y - 4z &= 0 \\ x + 3z &= 5 \end{aligned}$$

coefficient matrix $= \begin{pmatrix} 2 & 4 & 0 \\ -1 & 1 & -4 \\ 1 & 0 & 3 \end{pmatrix}$

variable vector $= \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ constant vector $= \begin{pmatrix} -7 \\ 0 \\ 5 \end{pmatrix}$

2) Find a solution for

$$\begin{aligned} 4x + 3y + 3z &= 2 \\ 3x + y + 2z &= 4 \\ -x - y - z &= -2 \end{aligned}$$

Hint:

$$\begin{pmatrix} 4 & 3 & 3 \\ 3 & 1 & 2 \\ -1 & -1 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ -2 & 1 & -5 \end{pmatrix}$$

$$Ax = b \quad \Rightarrow \quad x = A^{-1}b$$

$$\Rightarrow \begin{pmatrix} 4 & 3 & 3 \\ 3 & 1 & 2 \\ -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 \\ 1 & -1 & 1 \\ -2 & 1 & -5 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1(2) + 3(-2) \\ 1(2) + (-1)(4) + 1(-2) \\ -2(2) + 1(4) + (-5)(-2) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 10 \end{pmatrix} \Rightarrow \quad x = -4, \quad y = -4, \quad z = 10$$

5)

$$-x + 7y + 4z = 4$$
$$-3x \quad - z = 3$$
$$x + y + z = 1$$

Hint:

$$\begin{pmatrix} -1 & 7 & 4 \\ -3 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -3 & -7 \\ 2 & -5 & -13 \\ -3 & 8 & 21 \end{pmatrix}$$

$$A\underline{x} = b \quad \Rightarrow \quad \underline{x} = A^{-1}b$$

$$\begin{pmatrix} -1 & 7 & 4 \\ -3 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -3 & -7 \\ 2 & -5 & -13 \\ -3 & 8 & 21 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1(4) + (-3)(3) + (-7)(1) \\ 2(4) + (-5)(3) + (-13)(1) \\ (-3)(4) + 8(3) + (21)(1) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -12 \\ -20 \\ 33 \end{pmatrix} \Rightarrow \quad x = -12, \quad y = -20, \quad z = 33$$