Homework 8 Solutions

Linear Equations in Two Variables : 1,2,3,8,14

1) What are the coefficients of the equation $2x - 5y = -23$?

x coefficient $= 2$

y coefficient $= -5$

2) What is the constant of the equation $2x - 5y = -23$?

Constant $= -23$

3) Is $x = -4$, $y = 3$ a solution to the equation $2x - 5y = -23$?

$$2(-4) - 5(3) = -8 - 15 = -23 \checkmark$$

yes $x = -4$, $y = 3$ is a solution

8) Is $x = -1$ and $y = 3$ a solution to the system

$$7x + 2y = -1$$
$$5x - 3y = -14$$

$$7(-1) + 2(3) = -7 + 6 = -1 \checkmark$$
$$5(-1) - 3(3) = -5 - 9 = -14 \checkmark$$

yes, $x = -1$, $y = 3$ is a solution

14) Is there a unique solution to the system

$$① \quad 6x + 2y = 4$$
$$② \quad 15x + 5y = -7$$

$\Rightarrow$ ① $2y = 4 - 6x$

$y = 2 - 3x$ slope $= -3$, y-int $= 2$

② $5y = -7 - 15x$

$y = -7 - 3x$ slope $= -3$, y-int $= -7$

$\Rightarrow$ the 2 solutions are parallel therefore there is

no solution to the system.

Substitution: 3, 4

3) ① $3x = 5$

② $2x - 3y = 12$

solving ① for x we get
$$3x = 5$$
$$\boxed{x = \frac{5}{3}}$$

now plugging $x = \frac{5}{3}$ in to equation ② we get

$$2\left(\frac{5}{3}\right) - 3y = 12$$

$$\frac{10}{3} - 3y = 12$$

$$\frac{10}{9} - y = 4$$

$$-y = 4 - \frac{10}{9} = \frac{36}{9} - \frac{10}{9}$$

$$-y = \frac{26}{9}$$

$$\boxed{y = \frac{-26}{9}}$$

4) ① $2x + 8y = -8$

  ② $-3x + 6y = 12$

① $2x + 8y = -8$

  $2x = -8 - 8y$

  $x = -4 - 4y$

plug x into ②

  $-3(-4 - 4y) + 6y = 12$

  $12 + 12y + 6y = 12$

  $18y = 0$

  $\boxed{y = 0}$

plug y into ①

  $2x + 8(0) = -8$

  $2x = -8$

  $\boxed{x = -4}$

Linear Equations in Three Variables : 6 - 10

6)

  $3x - 4y + 2z = -9$

  $-4x + 4y + 10z = 32$

  $-x + 2y - 7z = -7$

Is $x = 1, y = 4,$ and $z = 2$ a solution?

  $3(1) - 4(4) + 2(2) = 3 - 16 + 4 = -9 \checkmark$

  $-4(1) + 4(4) + 10(2) = -4 + 16 + 20 = 32 \checkmark$

  $-(1) + 2(4) - 7(2) = -1 + 8 - 14 = -7 \checkmark$

Yes, $x = 1, y = 4$ and $z = 2$ is a solution

7)
$$3x - 4y + 2z = -9$$
$$-4x + 4y + 10z = 32$$
$$-x + 2y - 7z = -7$$

Is $x = -1$, $y = 1$, and $z = -1$ a solution?

$$3(-1) - 4(1) + 2(-1) = -3 - 4 - 2 = -9 \checkmark$$
$$-4(-1) + 4(1) + 10(-1) = 4 + 4 - 10 = -2 \neq 32$$

$\Rightarrow$ $x = -1$, $y = 1$, $z = -1$ is __not__ a solution

8) Is $x = 25$, $y = 23$, and $z = 4$ a solution?

$$3(25) - 4(23) + 2(4) = 75 - 92 + 8 = -9 \checkmark$$
$$-4(25) + 4(23) + 10(4) = -100 + 92 + 40 = 32 \checkmark$$
$$-(25) + 2(23) - 7(4) = -25 + 46 - 28 = -7 \checkmark$$

yes $x = 25$, $y = 23$ and $z = 4$ is a solution

9) Does the system have a unique solution?

No, both $(x = 1, y = 4, z = 2)$ and
$(x = 25, y = 23, z = 4)$ are solutions

10) Does the system have infinitely many solutions?

Yes, if there are at least 2 solutions then there are infinitely many