Homework 7 Solutions

Rational Functions: 1-3, 4-7

1) $\frac{4x^2-4}{x^3-3x^2+4} = \frac{4(x+1)(x-1)}{(x+1)(x-2)^2} = \frac{4(x-1)}{(x-2)^2} =$ **B**

Implied Domain = $\mathbb{R}-\{-1, 2\}$

2) $\frac{x^3-3x^2+4}{2x-4} = \frac{(x+1)(x-2)^2}{2(x-2)} = \frac{1}{2}(x+1)(x-2) =$ **A**

Implied Domain = $\mathbb{R}-\{2\}$

3) $\frac{2x-4}{4x^2-4} = \frac{2(x-2)}{4(x+1)(x-1)} = \frac{(x-2)}{2(x+1)(x-1)} =$ **C**

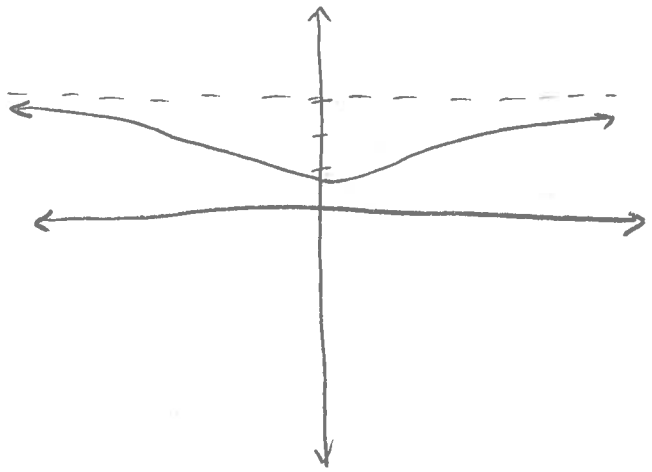
Implied Domain = $\mathbb{R}-\{1, -1\}$

4) $\frac{3(x^2+1)}{x^2+5}$

roots = none

Vertical asymptotes = none

L.O.T = $\frac{3x^2}{x^2} = 3$ $\Rightarrow$ horizontal asymptote at 3

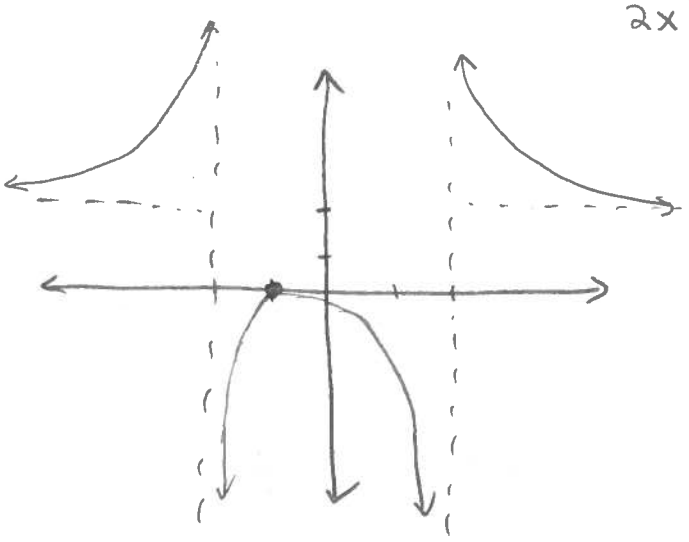


5) $\frac{4(x+1)^2}{2(x+2)(x-2)}$

roots = -1 (double root)

Vertical asymptotes = $2, -2$

L.O.T. = $\frac{4x^2}{2x^2} = 2$ $\Rightarrow$ horizontal asymptote @ 2

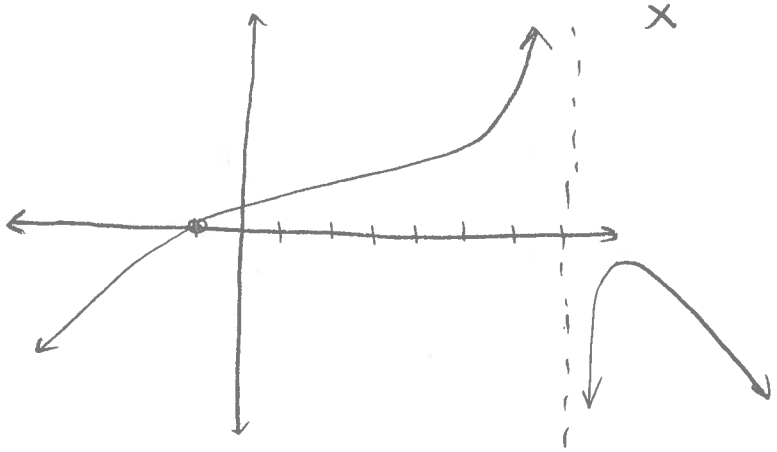


6) $\frac{-(x+1)(x^2+1)(x^2+8)}{(x-7)}$

roots: -1

vertical asymptotes = 7

LOT = $\frac{-x^5}{x} = -x^4$

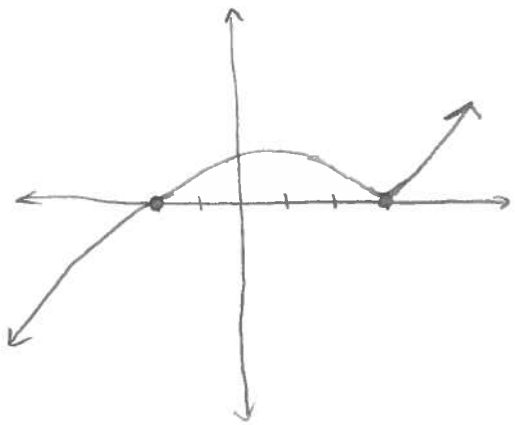


7) $7(x+2)^3(x-3)^2$

roots = -2 (triple), 3 (double)

Vertical asymptotes = none

LOT = $7(x^3)(x^2) = 7x^5$



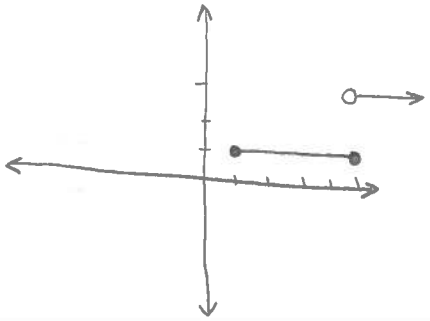
Piecewise Defined Functions: 2, 4, 12

2) $g(x) = \begin{cases} 3 & \text{if } x \in [1,5] \\ 1 & \text{if } x \in (5,\infty) \end{cases}$

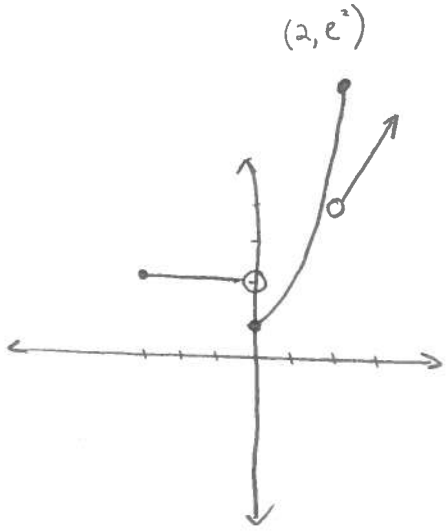
$g(1) = 3$ b/c $1 \in [1,5]$

$g(100) = 1$ b/c $100 \in (5,\infty)$

$g(5) = 3$ b/c $5 \in [1,5]$



4) $f(x) = \begin{cases} 2 & \text{if } x \in [-3,0) \\ e^x & \text{if } x \in [0,2] \\ 3x-2 & \text{if } x \in (2,\infty) \end{cases}$



$f(-2) = 2$ b/c $-2 \in [-3,0)$

$f(0) = e^0 = 1$ b/c $0 \in [0,2]$

$f(2) = e^2$ b/c $2 \in [0,2]$

$f(15) = 3(15) - 2 = 43$ b/c $15 \in (2,\infty)$

12) Solve for x if $|3x+4| < 1$

$$|3x+4| = \begin{cases} 3x+4 & \text{if } x \geq \frac{-4}{3} \\ -3x-4 & \text{if } x \leq \frac{-4}{3} \end{cases}$$

$\Rightarrow$ if $x \geq \frac{-4}{3}$ $\quad |3x+4| = 3x+4 < 1$

$3x < -3 \quad \Rightarrow \quad \frac{-4}{3} \leq x < -1$

$x < -1$

if $x \leq \frac{-4}{3}$ $\quad |3x+4| = -3x-4 < 1$

$-3x < 5 \quad \Rightarrow \quad \frac{-5}{3} < x \leq \frac{-4}{3}$

$x > \frac{-5}{3}$

combining the 2 intervals we get

$$\boxed{\frac{-5}{3} < x < -1}$$

Additional Problems:

1) Does the rational function $r(x) = \dfrac{(x+3)(x-2)}{(x-2)}$ exactly equal the polynomial $p(x) = x+3$?

No, if $x = 2$ $\quad p(x) = 5$ but $r(x)$ is undefined

$\Rightarrow \quad p(x) \neq r(x)$

2) Are all polynomial functions also rational functions?

Yes, let the polynomial function be $p(x)$

then $p(x) = \frac{p(x)}{1} = r(x)$

(just divide by 1)

Are all rational functions also polynomial functions?

No, consider $r(x) = \frac{1}{x} = x^{-1}$ is not a polynomial, the exponents of a polynomial are natural numbers. $-1 \notin \mathbb{N}$