Homework 6 Solutions

Exponential Functions: 5-11, 16-18, 23, 24

5) $4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{\sqrt{4^3}} = \frac{1}{\sqrt{(2^2)^3}} = \frac{1}{\sqrt{2^6}} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$ or 2^{-3}

6) $\left(3^{200}\right)^{\frac{1}{100}} = 3^{200 \cdot \frac{1}{100}} = 3^2 = \boxed{9}$

7) $1000^{\frac{2}{3}} = \left(10^3\right)^{\frac{2}{3}} = 10^{3 \cdot \frac{2}{3}} = 10^2 = \boxed{100}$

8) $97^{-16}\,97^{15} = 97^{(-16+15)} = 97^{-1} = \boxed{\frac{1}{97}}$

9) $36^{\frac{3}{6}} = \left(6^2\right)^{\frac{3}{6}} = 6^{\frac{6}{6}} = 6^1 = \boxed{6}$

10) $\frac{3^{297}}{3^{300}} = 3^{(297-300)} = 3^{-3} = \frac{1}{3^3} = \boxed{\frac{1}{27}}$

11) $\left(5^{\frac{4}{7}}\right)^{\frac{14}{4}} = 5^{\frac{4 \cdot 14}{4 \cdot 7}} = 5^2 = \boxed{25}$

16) $4^x = 16 = 4^2 \Rightarrow \boxed{x = 2}$

17) $2^x = 8 = 2^3 \Rightarrow \boxed{x = 3}$

18) $10^x = 10{,}000 = 10^4 \Rightarrow \boxed{x = 4}$

23) $8^x = \frac{1}{4}$

$(2^3)^x = \frac{1}{2^2} = 2^{-2}$

$\Rightarrow 2^{3x} = 2^{-2}$ $\boxed{x = \frac{-2}{3}}$

24) $27^x = \frac{1}{9}$

$(3^3)^x = \frac{1}{3^2} = 3^{-2}$

$\Rightarrow 3^{3x} = 3^{-2}$ $\boxed{x = \frac{-2}{3}}$

Logarithms: 13-18, 30-34

13) $\log_4(16) = \log_4(4^2) = 2\log_4(4) = \boxed{2}$

14) $\log_2(8) = \log_2(2^3) = 3\log_2(2) = \boxed{3}$

15) $\log_{10}(10{,}000) = \log_{10}(10^4) = 4\log_{10}(10) = \boxed{4}$

16) $\log_3(9) = \log_3(3^2) = 2\log_3(3) = \boxed{2}$

17) $\log_5(125) = \log_5(5^3) = 3\log_5(5) = \boxed{3}$

18) $\log_{\frac{1}{2}}(16) = \log_{\frac{1}{2}}(2^4) = \log_{\frac{1}{2}}\left(\left[\left(\frac{1}{2}\right)^{-1}\right]^4\right) = \log_{\frac{1}{2}}\left(\frac{1}{2}^{-4}\right)$

$= -4\log_{\frac{1}{2}}\left(\frac{1}{2}\right) = \boxed{-4}$

30) $\log_4(x) = -2$

$$4^{\log_4(x)} = 4^{-2}$$

$$x = 4^{-2} = \frac{1}{4^2} = \boxed{\frac{1}{16}}$$

31) $\log_6(x) = 2$

$$6^{\log_6(x)} = 6^2$$

$$\boxed{x = 36}$$

32) $\log_3(x) = -3$

$$3^{\log_3(x)} = 3^{-3}$$

$$x = 3^{-3} = \frac{1}{3^3} = \boxed{\frac{1}{27}}$$

33) $\log_{\frac{1}{10}}(x) = -5$

$$\frac{1}{10}^{\log_{\frac{1}{10}}(x)} = \left(\frac{1}{10}\right)^{-5}$$

$$x = \left(\frac{1}{10}\right)^{-5} = \frac{1}{\left(\frac{1}{10}\right)^5} = \frac{1}{\frac{1}{100000}} = \boxed{100000}$$

34) $e^x = 17$

$$\log_e(e^x) = \log_e(17) \Rightarrow x\log_e(e) = \log_e(17)$$

$$\boxed{x = \log_e(17)}$$

Exponential and Logarithmic Equations: 1-6, 10-14

1) $10^{3x} = 1000$

$\log_{10}(10^{3x}) = \log_{10}(1000) = \log_{10}(10^3)$

$3x = 3$

$\boxed{x = 1}$

2) $6(14^x) = 30$

$14^x = 5$

$\log_{14}(14^x) = \log_{14}(5)$

$\boxed{x = \log_{14}(5)}$

3) $2e^x = 8$

$e^x = 4$

$\log_e(e^x) = \log_e(4)$

$\boxed{x = \log_e(4)}$

4) $e^x + 10 = 17$

$e^x = 7$

$\log_e(e^x) = \log_e(7)$

$\boxed{x = \log_e(7)}$

5) $(3^x)^5 = 27$

$3^{5x} = 27$

$\log_3(3^{5x}) = \log_3(27) = \log_3(3^3) = 3$

$5x = 3$

$\boxed{x = \frac{3}{5}}$

6) $5^{\frac{-x}{2}} = \frac{1}{5}$

$\log_5\left(5^{\frac{-x}{2}}\right) = \log_5\left(\frac{1}{5}\right) = \log_5(5^{-1}) = -1$

$\frac{-x}{2} = -1 \quad \Rightarrow \quad \boxed{x = 2}$

10) $\log_3(x-5) = 2$

$3^{\log_3(x-5)} = 3^2 = 9$

$x - 5 = 9$

$\boxed{x = 14}$

11) $\log_e(x) = -6$

$e^{\log_e(x)} = e^{-6}$

$\boxed{x = e^{-6} = \frac{1}{e^6}}$

12) $\log_e(2x) = 24$

$$e^{\log_e(2x)} = e^{24}$$

$$2x = e^{24}$$

$$\boxed{x = \frac{e^{24}}{2}}$$

13) $\log_e(\sqrt{x-4}) = 5$

$$\log_e((x-4)^{\frac{1}{2}}) = 5$$

$$\frac{1}{2}\log_e(x-4) = 5$$

$$\log_e(x-4) = 10$$

$$e^{\log_e(x-4)} = e^{10}$$

$$x-4 = e^{10}$$

$$\boxed{x = e^{10}+4}$$

14) $\log_2(x^7) = 28$

$$7\log_2(x) = 28$$

$$\log_2(x) = 4$$

$$2^{\log_2(x)} = 2^4$$

$$x = 2^4 \Rightarrow \boxed{x = 16}$$