Homework 4 Solutions

Basics of Polynomials: 1-3, 6-8, 13-16, 19-23

1) Degree: 3, Leading Coefficient: 2, Leading Order Term: $2x^3$

2) Degree: 2, Leading Coefficient: -1, Leading Order Term: $-x^2$

3) Degree: 1, Leading Coefficient: 1, Leading Order Term: x

6) $(2x+3)+(-x+5)$

$$= 2x - x + 3 + 5$$

$$= \boxed{x+8}$$

7) $(3x^2 - x + 6) - (3x^2 + x - 6)$

$$= 3x^2 - 3x^2 - x - x + 6 - (-6)$$

$$= \boxed{-2x+12}$$

8) $(8x^2 - 5x - 2) + (4x^5 - x^2 + 3x + 7)$

$$= 4x^5 + 8x^2 - x^2 - 5x + 3x - 2 + 7$$

$$= \boxed{4x^5 + 7x^2 - 2x + 5}$$

13) $6x^2(3x+1)$

$$= 6x^2(3x) + 6x^2(1)$$

$$= \boxed{18x^3 + 6x^2}$$

14) $2x(x^3+4x-6)$

$= 2x(x^3) + 2x(4x) + 2x(-6)$

$= \boxed{2x^4 + 8x^2 - 12x}$

15) $(x^2+6)(x-5)$

$= x^2(x) + x^2(-5) + 6(x) + 6(-5)$

$= \boxed{x^3 - 5x^2 + 6x - 30}$

16) $(5x^3+8)(x^2+2x-1)$

$= 5x^3(x^2) + 5x^3(2x) + 5x^3(-1) + 8(x^2) + 8(2x) + 8(-1)$

$= \boxed{5x^5 + 10x^4 - 5x^3 + 8x^2 + 16x - 8}$

19) Leading Order Term of:

$3(7x^4 - x^3 + 5x^2 - 13x + 3)(4x^5 - 6x^2 - 5x)$

$= 3(7x^4)(4x^5) = \boxed{84x^9}$

20) Leading Order term of:

$2(x+1)$

$= 2(x) = \boxed{2x}$

21) Leading Order term of:

$$-5(x+4)(x-5)$$

$$= -5(x)(x) = \boxed{-5x^2}$$

22) Leading order term of:

$$8(x-3)(x-5)(x-6)(x-9)$$

$$= 8(x)(x)(x)(x) = \boxed{8x^4}$$

23) Leading order term of:

$$-3(x+3)(x^2-4x+2)$$

$$= -3(x)(x^2) = \boxed{-3x^3}$$

Polynomial Division: 5-10

5)

$$\begin{array}{r|l} & 3x^2 - 2x - 4 \\ \hline 4x^2-2 & 12x^4 - 8x^3 - 22x^2 + 4x + 8 \\ & -(12x^4 + 0x^3 - 6x^2) \\ \hline & -8x^3 - 16x^2 + 4x \\ & -(-8x^3 + 0x^2 + 4x) \\ \hline & -16x^2 + 0x + 8 \\ & -(-16x^2 + 0x + 8) \\ \hline & 0 \end{array}$$

Answer = $\boxed{3x^2 - 2x - 4}$

6) $\quad -x + 14$

$x-1 \,\big)\overline{-x^2 + 15x - 1}$

$-\ (-x^2 + x)$

$14x - 1$

$-\ (14x - 14)$

13 = remainder

Answer: $\boxed{-x + 14 + \frac{13}{x-1}}$

7) $\quad x^2 + 5x + 6$

$x-1 \,\big)\overline{x^3 + 4x^2 + x - 6}$

$-\ (x^3 - x^2)$

$5x^2 + x$

$-\ (5x^2 - 5x)$

$6x - 6$

$-\ (6x - 6)$

0

Answer: $\boxed{x^2 + 5x + 6}$

8) $\quad x^2 - 3x + 4$

$x^2+1 \,\big)\overline{x^4 - 3x^3 + 5x^2 - 2x + 9}$

$-\ (x^4 + 0x^3 + x^2)$

$-3x^3 + 4x^2 - 2x$

$-\ (-3x^3 + 0x^2 - 3x)$

$4x^2 + x + 9$

$-\ (4x^2 + 0x + 4)$

$x + 5$ = remainder

Answer: $\boxed{x^2 - 3x + 4 + \frac{x+5}{x^2+1}}$

9) $$\begin{array}{r} 4x^2 - 21x + 104 \\ x+5 \overline{\smash{)}\, 4x^3 - x^2 - x - 1} \end{array}$$

$-(4x^3 + 20x^2)$

$-21x^2 - x$

$-(-21x^2 - 105x)$

$104x - 1$

$-(104x + 520)$

-521 = remainder

Answer: $\boxed{4x^2 - 21x + 104 - \frac{521}{x+5}}$

10) $$\begin{array}{r} 6x^4 + 23x^3 + 69x^2 + 205x + 665 \\ x-3 \overline{\smash{)}\, 6x^5 + 5x^4 - 2x^2 + 50x - 13} \end{array}$$

$-(6x^5 - 18x^4)$

$23x^4 + 0x^3$

$-(23x^4 - 69x^3)$

$69x^3 - 2x^2$

$-(69x^3 - 207x^2)$

$205x^2 + 50x$

$-(205x^2 - 615x)$

$665x - 13$

$-(665x - 1995)$

1982 = remainder

Answer: $\boxed{6x^4 + 23x^3 + 69x^2 + 205x + 665 + \frac{1982}{x-3}}$

Roots and Factors : 1-6, 8-14

1) $(x-1)(x-2)$ $\boxed{\text{Roots} = 1, 2}$

2) $-(x+7)(x-3)(x^4+x^3+2x^2+x+1)$

$\boxed{\text{Roots} = -7, 3}$

3) $-\frac{2}{5}(x+\frac{7}{3})(x+\frac{1}{2})(x-\frac{4}{3})(x-\frac{9}{2})(x^2+1)$

$\boxed{\text{Roots} = -\frac{7}{3}, -\frac{1}{2}, \frac{4}{3}, \frac{9}{2}}$

4) x^3+4x-5 has root $x = 1$

$(1)^3+4(1)-5 = 1+4-5 = 0$

$\Rightarrow$ factor = $(x-1)$

$$\begin{array}{r|l}
 & x^2+x+5 \\ \hline
x-1 & x^3+4x-5 \\
 & -(x^3-x^2) \\ \cline{2-2}
 & \quad x^2+4x \\
 & \quad -(x^2-x) \\ \cline{2-2}
 & \qquad 5x-5 \\
 & \qquad -(5x-5) \\ \cline{2-2}
 & \qquad\quad 0
\end{array}$$

$x^3+4x-5 = \boxed{(x-1)(x^2+x+5)}$

5) $x^3 + x$ has root $x = 0$

$(0)^3 + (0) = 0$ thus $(x - 0) = x$ is a factor

$$x^3 + x = \boxed{x(x^2 + 1)}$$

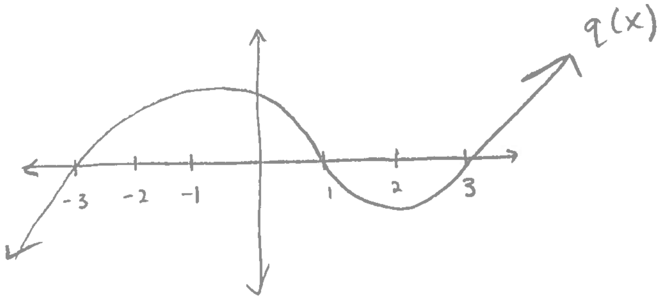
6) $x^5 + 3x^4 + x^3 - x^2 - x - 1$ has root $x = -1$

$(-1)^5 + 3(-1)^4 + (-1)^3 - (-1)^2 - (-1) - 1 = 0$ thus $(x+1)$ is a factor

$$\begin{array}{r|l}
 & x^4 + 2x^3 - x^2 - 1 \\ \hline
x+1 & x^5 + 3x^4 + x^3 - x^2 - x - 1 \\
 & -(x^5 + x^4) \\ \hline
 & 2x^4 + x^3 \\
 & -(2x^4 + 2x^3) \\ \hline
 & -x^3 - x^2 \\
 & -(-x^3 - x^2) \\ \hline
 & 0 - x - 1 \\
 & -(-x - 1) \\ \hline
 & 0
\end{array}$$

Thus $x^5 + 3x^4 + x^3 - x^2 - x - 1 = \boxed{(x+1)(x^4 + 2x^3 - x^2 - 1)}$

8)



roots = x-intercepts = $-3, 1, 3$

factors = $(x+3), (x-1), (x-3)$

9) $(x+3)(x-2)$

Leading Order term = $(x)(x) = x^2$

$\Rightarrow$ Degree = 2

10) $(3x+5)(4x^2+2x-3)$

Leading order term = $(3x)(4x^2) = 12x^3$

$\Rightarrow$ Degree = 3

11) $-17(3x^2+20x-4)$

Leading order term = $(-17)(3x^2) = -51x^2$

$\Rightarrow$ Degree = 2

12) $4(x-1)(x-1)(x-1)(x-2)(x^2+7)(x^2+3x-4)$

Leading order term = $4(x)(x)(x)(x)(x^2)(x^2)$

$= 4x^8$ $\Rightarrow$ Degree = 8

13) $5(x-3)(x^2+1)$

Leading order term = $5(x)(x^2) = 5x^3$

Degree = 3

14) False, $7x^5 + 13x^4 - 3x^3 - 7x^2 + 2x - 1$ is

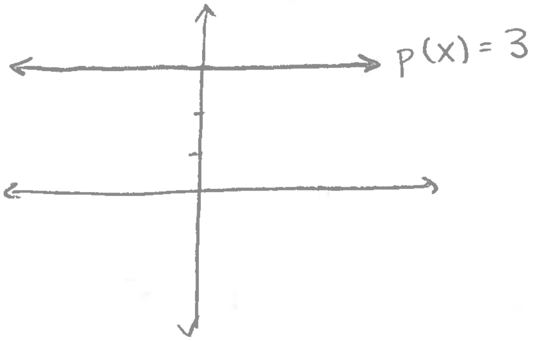
degree 5, # roots $\leq$ degree

$\Rightarrow$ # roots ≤ 5

but $8 \nleq 5$

<u>Constant</u> <u>and</u> <u>Linear</u> <u>Polynomials</u> : 1, 4, 6, 7

1)

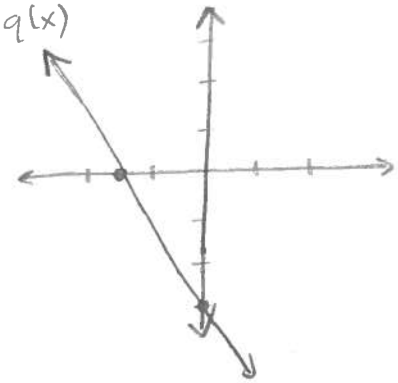


4) $q(x) = \frac{2}{9}x - \frac{8}{5} = 0$

$$\frac{2}{9}x = \frac{8}{5}$$

$$x = \frac{8 \cdot 9}{2 \cdot 5} = \frac{4 \cdot 9}{5} = \boxed{\frac{36}{5}}$$

6)



$q(x) = -2x - 3$

y-intercept = $b = -3$

root = x-int = $\frac{-b}{a} = \frac{-(-3)}{-2} = \frac{-3}{2}$

7) $p(x) = 3(x) - 200$

earnings per coconut

number of coconuts

initial loss from license cost

Quadratic Polynomials: 1-3

1) $-2x^2 - 2x + 12$

a) $a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$ $= -2\left(x + \frac{-2}{2(-2)}\right)^2 + 12 - \frac{(-2)^2}{(4)(-2)}$

$a = -2, \quad b = -2, \quad c = 12$ $= -2\left(x + \frac{1}{2}\right)^2 + \frac{25}{2}$

b) Vertex $= \left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$

$= \left(\frac{-1}{2}, \frac{25}{2}\right)$

c) leading order coefficient is -2 ⇒ parabola opens down

d) Discriminant $= b^2 - 4ac = (-2)^2 - (4)(-2)(12)$

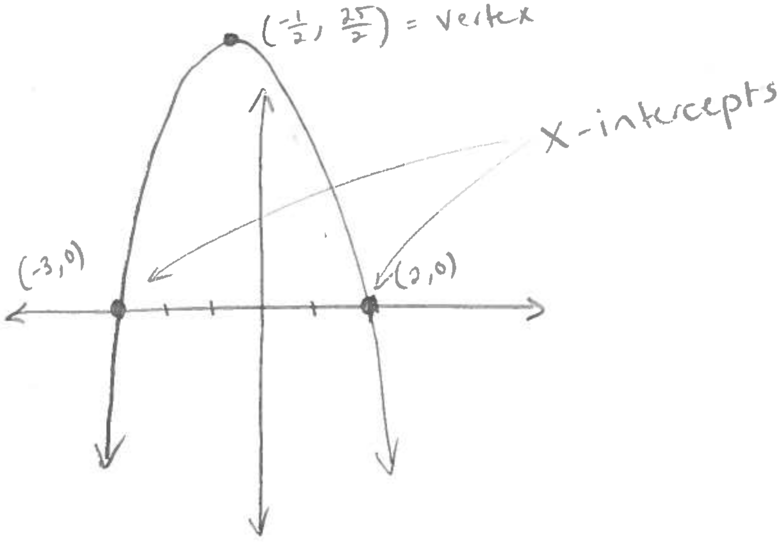
$= 4 + 96 = 100$

e) Dicriminant > 0 ⇒ 2 roots

f) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{100}}{2(-2)} = \frac{2 \pm 10}{-4}$

$X = -3, 2$

g)



2) $x^2 + 2x + 1$

a) $a(x+\frac{b}{2a})^2 + c - \frac{b^2}{4a} = 1(x+1)^2 + 1 - \frac{4}{4(1)}$

$a=1, \ b=2, \ c=1$ $\qquad = \boxed{(x+1)^2}$

b) vertex $= (\frac{-b}{2a}, c - \frac{b^2}{4a})$

$= (-1, 0)$

c) Leading order coefficient $= 1 > 0$

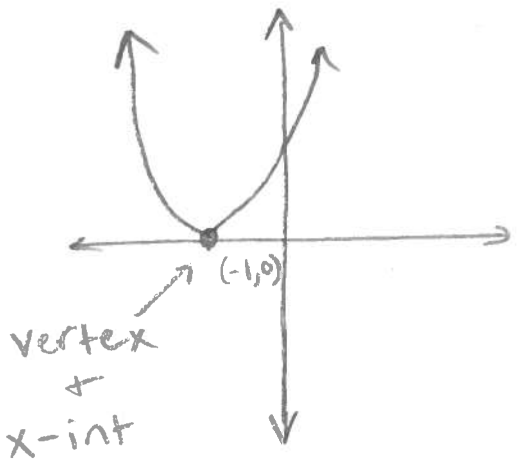
$\Rightarrow$ parabola opens up

d) Discriminant $= b^2 - 4ac = (2)^2 - (4)(1)(1) = 0$

e) Discriminant $= 0 \Rightarrow 1$ double root

f) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(2) \pm \sqrt{0}}{2(1)} = \boxed{-1}$

g)



3) $3x^2 - 9x + 6$

a) $a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 3\left(x + \frac{-9}{2(3)}\right)^2 + 6 - \frac{(-9)^2}{4(3)}$

$a = 3,\ b = -9,\ c = 6$

$$= 3\left(x - \frac{3}{2}\right)^2 + 6 - \frac{27}{4}$$

$$= 3\left(x - \frac{3}{2}\right)^2 - \frac{3}{4}$$

b) Vertex = $\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$

$$= \left(\frac{3}{2}, \frac{-3}{4}\right)$$

c) Leading order coefficient = $3 > 0$ ⇒ opening up

d) Discriminant = $b^2 - 4ac = (-9)^2 - 4(3)(6)$

$$= 81 - 72 = 9$$

e) Discriminant = $9 > 0$ ⇒ 2 roots

f) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9) \pm \sqrt{9}}{2(3)} = \frac{9 \pm 3}{6}$

$X = 1, 2$

g)