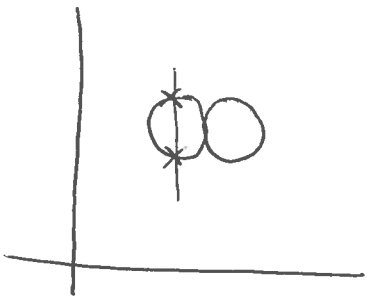
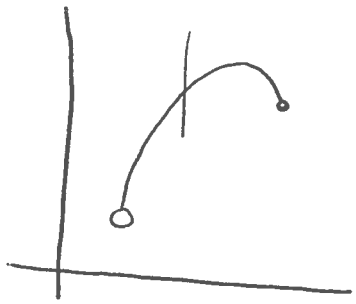
Homework 3 Solutions

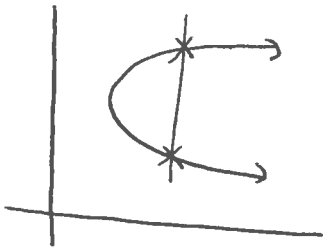
Intro to Graphs: (1-12)

1) 

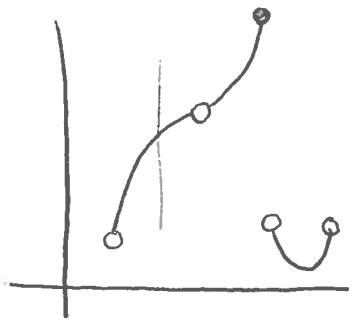
not a graph of a function, fails the vertical line test

2) 

Is a graph of a function, passes the vertical line test

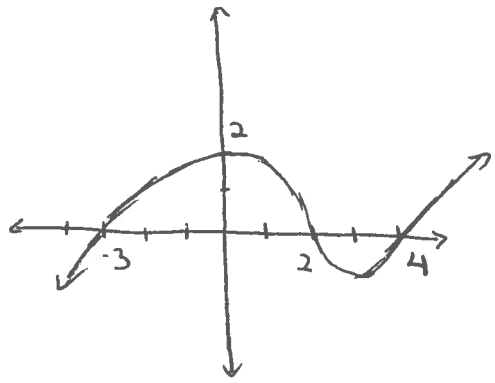
3) 

not a graph of a function, ~~passes~~ fails the vertical line test

4) 

Is a graph of a function, passes the vertical line test

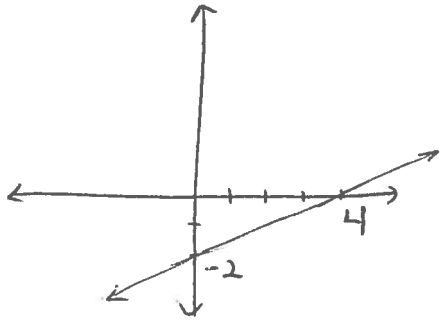
5)



X-ints: $(-3,0)$, $(2,0)$, $(4,0)$

y-ints: $(0,2)$

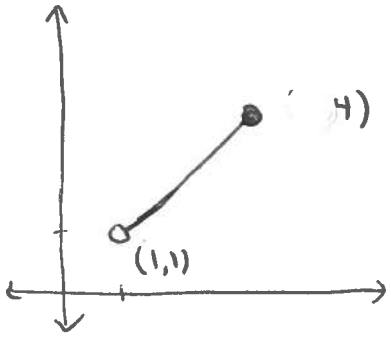
6)



X-ints: $(4,0)$

y-ints: $(0,-2)$

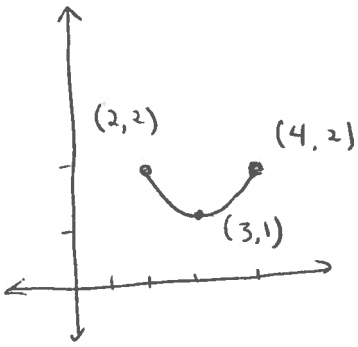
7)



Domain = $(1, 3]$

Range = $(1, 4]$

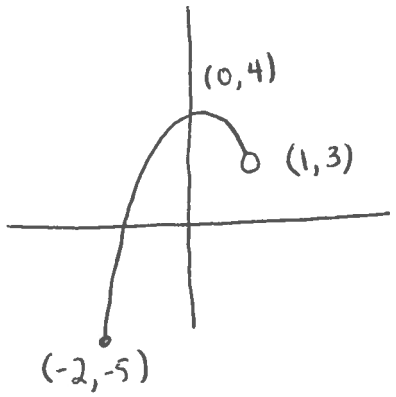
8)



Domain = $[2,4]$

Range = $[1,2]$

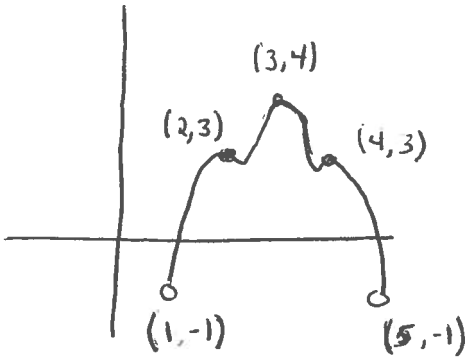
9)



Domain = $[-2, 3)$

Range = $[-5, 4]$

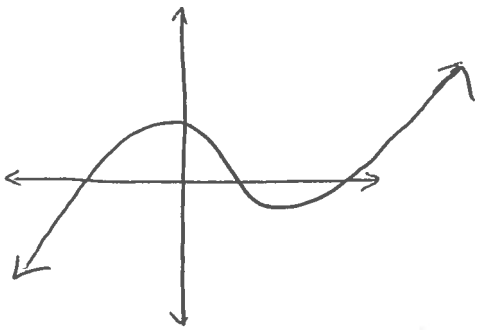
10)



Domain = $(1, 5)$

Range = $(-1, 4]$

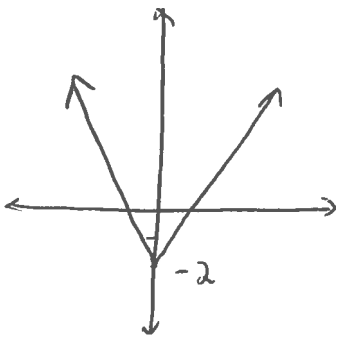
11)



Domain = $\mathbb{R}$

Range = $\mathbb{R}$

12)



Domain = $\mathbb{R}$

Range = $[-2, \infty)$

Graph Transformations: 1-16

1) $f(x)+2 = x^8+2$ D

2) $3f(x) = 3x^8 =$ I

3) $f(-x) = (-x)^8 = x^8$ A

4) $f(x-2) = (x-2)^8$ G

5) $\frac{1}{3}f(x) = \frac{1}{3}x^8$ B

6) $f(3x) = (3x)^8$ H

7) $f(x)-2 = x^8-2$ C

8) $-f(x) = -x^8$ F

9) $f(x+2) = (x+2)^8$ J

10) $f(\frac{x}{3}) = (\frac{x}{3})^8$ E

11) $-4g(3x-7)+2 = -4\left(\frac{1}{3x-7}\right)+2$

$= \frac{-4}{3x-7}+2$ B

12) $6g(-2x+5)-3 = 6\left(\frac{1}{-2x+5}\right)-3$

$= \frac{6}{-2x+5}-3$ A

13) $f(x)+2$ After transformation of $\uparrow 2$

$\Rightarrow (4,1) \longmapsto (4,3)$ **A**

14) $f(x)-2$ After transformation of $\downarrow 2$

$(4,1) \longmapsto (4,-1)$ **D**

15) $f(x+2)$ Before transformation of $\leftarrow 2$

$(4,1) \longmapsto (2,1)$ **B**

16) $f(x-2)$ Before transformation of $\rightarrow 2$

$(4,1) \longmapsto (6,1)$ **C**

Inverse Functions: (1-3, 7-9, 15-20)

1) $g(2) = 3$ therefore $g^{-1}(3) = 2$

2) $g(7) = -2$ therefore $g^{-1}(-2) = 7$

3) $g(-10) = 5$ therefore $g^{-1}(5) = -10$

Using the chart: $(f^{-1}(5) = -7,\ f^{-1}(3) = 2,\ f^{-1}(1) = -2)$

7) $f(x+2) = 5$

$\Rightarrow f^{-1}(5) = (x+2) = -7$

$\boxed{x = -9}$

8) $f(3x-4) = 3$

$\Rightarrow \quad 3x - 4 = f^{-1}(3) = 2$

$3x = 6 \qquad \boxed{x = 2}$

9) $f(-5x) = 1$

$\Rightarrow \quad -5x = f^{-1}(1) = -2$

$x = \frac{-2}{-5} \quad \Rightarrow \quad \boxed{x = \frac{2}{5}}$

15) $h(x) = \frac{1}{x}$

$\Rightarrow \quad y = \frac{1}{x}$

$xy = 1$

$x = \frac{1}{y} = h^{-1}(y) \quad \Rightarrow \quad \boxed{h^{-1}(x) = \frac{1}{x}}$

16) $f(x) = \frac{x}{x-1}$

$\Rightarrow \quad y = \frac{x}{x-1}$

$y(x-1) = x$

$xy - y = x$

$-y = x - xy$

$-y = x(1-y)$

$x = \frac{-y}{1-y} = f^{-1}(y)$

$\Rightarrow \boxed{f^{-1}(x) = \frac{-x}{1-x} = \frac{x}{x-1}}$

17) $g(x) = \dfrac{2x+3}{x}$

$\Rightarrow y = \dfrac{2x+3}{x}$

$xy = 2x + 3$

$xy - 2x = 3$

$x(y-2) = 3$

$x = \dfrac{3}{y-2} = g^{-1}(y)$

$\Rightarrow \boxed{g^{-1}(x) = \dfrac{3}{x-2}}$

18) $h(x) = \dfrac{x}{4-x}$

$\Rightarrow y = \dfrac{x}{4-x}$

$y(4-x) = x$

$4y - xy = x$

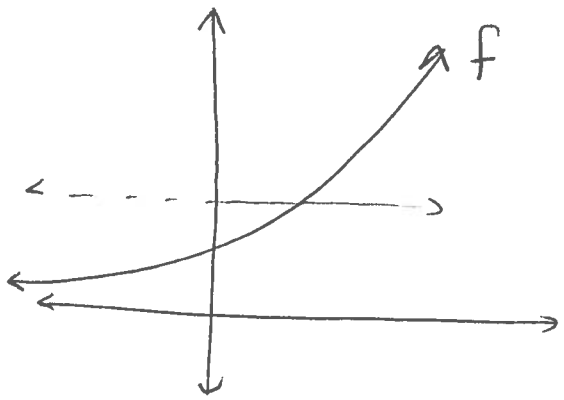
$4y = x + xy$

$4y = x(1+y)$

$\dfrac{4y}{1+y} = x = h^{-1}(y)$

$\boxed{h^{-1}(x) = \dfrac{4x}{1+x}}$

19) $f: \mathbb{R} \to (0, \infty)$



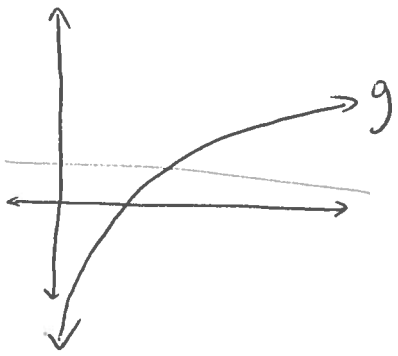
f passes the horizontal line test $\Rightarrow$ f is one to one

range = $(0, \infty)$ = target

$\Rightarrow$ f is onto

Since f is both one-to-one and onto, f has an inverse

20) $g: (0, \infty) \to \mathbb{R}$. Does g have an inverse?



g passes the horizontal line test

$\Rightarrow$ g is one-to-one

range = $\mathbb{R}$ = target therefore g is onto

g is one-to-one and onto, g has an inverse

Other Problems:

1) A graph of a function can have infinitely many x-intercepts (consider $f(x) = 0$)

   A graph of a function may only have at most one y-intercept (if it had more than one it would fail the vertical line test)

2) A graph may fail the vertical line test, a graph of a function may not.