

Math 1050-006 Homework 2 Solutions

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1 Book Problems

1.1 Counting 1

1. First let us pick the first number in the PIN code. As there is no restriction on the first number it can be any of the 10 digits (0-9). Now when we pick the second number however we do have one condition, it can't be the same as the first number, so there are only 9 choices. The same will be true for the remaining 3 choices, the third can't be the same as the second, the fourth can't be the same as the third, and the fifth can't be the same as the fourth. Now that the no adjacent numbers can be the same condition has been taken care of, we can use options multiply to find that the total number of five-digit pins with no adjacent numbers the same is equal to

$$10 * 9 * 9 * 9 * 9 = 10 * 9^4 = 10 * 81^2 = 65610$$

2. Making an outfit is like making a sandwich, all of your individual choices for shoes, pants, shirts, jackets, and hats are all independent. Thus as there are 3 shoes, 4 pants, 6 shirts, 2 jackets, and 2 hats we can use options multiply to get:

$$3 * 4 * 6 * 2 * 2 = 12 * 12 * 2 = 144 * 2 = 288$$

3. The intent of this problem is that each of the letters can only be used once in the arrangement. As such we can use any of the 5 letters for the first spot, any of the 4 remaining letters for the second spot, each of the 3 remaining letters for the 3rd spot, either of the 2 remaining letters in the 4th spot, and then the last letter in the last spot. Using options multiply, (or the formula for arranging a set of 5 objects, 5!) we get:

$$5 * 4 * 3 * 2 * 1 = 5! = 120$$

6. Any of the 51 contestants in the beauty pageant could win, however once a winner has been chosen only 1 of the remaining contestants could be runner up, and finally only one of the remaining 49 contestants could win the congeniality award. Using options multiply we are left with:

$$51 * 50 * 49 = 124950$$

7. This problem is the same as the beauty pageant problem. Any of the 10 people on the boat could be the captain, then once the captain is

chosen any of the 9 remaining people can be picked as first mate, and finally any of the 8 remaining people can be chosen to be the deck swab. Using options multiply we are left with:

$$10 * 9 * 8 = 720$$

8. Any of the 50 states can be the most desirable, then any of the 49 could be second most desirable, then any of the remaining 48 could be third most desirable, and any of the remaining 47 can be 50th most desirable. Continuing we have that any of the remaining 46 can be 49th most desirable and finally any of the remaining 45 can be the 48th most desirable. Using options multiply we get:

$$50 * 49 * 48 * 47 * 46 * 45 = 11441304000$$

1.2 Counting 2

1. There are 7 colors of which we want to pick 3 of them. The order in which we pick the colors does not matter. Therefore we have:

$$\binom{7}{3} = \frac{7!}{(7-3)!(3)!} = \frac{7!}{4! * 3!} = \frac{7 * 6 * 5}{3 * 2 * 1} = 7 * 5 = 35$$

2. The order in which you purchase the 7 tickets for the 15 games does not matter so we get:

$$\begin{aligned} \binom{15}{7} &= \frac{15!}{(15-7)!(7)!} = \frac{15!}{7! * 8!} = \frac{15 * 14 * 13 * 12 * 11 * 10 * 9}{7 * 6 * 5 * 4 * 3 * 2 * 1} \\ &= 3 * 13 * 11 * 5 * 3 = 6435 \end{aligned}$$

4. For this lottery we are not told that order matters, so we can assume that order does not matter. As such there are 59 numbers of which we are picking 6, or 59 choose 6 ways to make a lottery ticket.

$$\begin{aligned} \binom{59}{6} &= \frac{59!}{(59-6)!(6)!} = \frac{59!}{53! * 6!} = \frac{59 * 58 * 57 * 56 * 55 * 54}{6 * 5 * 4 * 3 * 2 * 1} \\ &= 3 * 11 * 7 * 59 * 58 * 57 = 45057474 \end{aligned}$$

5. We need the n=6, or 7th row of Pascal's triangle, which is:

$n = 0:$				1								
$n = 1:$			1		1							
$n = 2:$			1		2		1					
$n = 3:$			1		3		3	1				
$n = 4:$		1		4		6		4	1			
$n = 5:$	1		5		10		10		5	1		
$n = 6:$	1		6		15		20		15		6	1

We now can use the binomial theorem:

$$(x + y)^n = \sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$$

Which for this problem with $n=6$ and using the binomial coefficients from Pascal's triangle gives us:

$$(x+y)^6 = \sum_{i=0}^6 \binom{6}{i} x^i y^{6-i} = y^6 + 6xy^5 + 15x^2y^4 + 20x^3y^3 + 15x^4y^2 + 6x^5y + x^6$$

7. Using the binomial theorem with $n=3$, and the $n=3$ row of Pascal's triangle, and finally setting $y=2$ we get:

$$(x+2)^3 = \sum_{i=0}^3 \binom{3}{i} x^i 2^{3-i} = 2^3 + 3x(2)^2 + 3x^2(2) + x^3 = 8 + 12x + 6x^2 + x^3$$

8. Similarly use the binomial theorem with $n=4$ and the $n=4$ row of Pascal's triangle, this time using $x=2x$, $y=-1$ we get:

$$\begin{aligned} (2x - 1)^4 &= \sum_{i=0}^4 \binom{4}{i} (2x)^i (-1)^{4-i} = \\ &(-1)^4 + 4(2x)(-1)^3 + 6(2x)^2(-1)^2 + 4(2x)^3(-1) + (2x)^4 = \\ &1 - 8x + 24x^2 - 32x^3 + 16x^4 \end{aligned}$$

1.3 More on Functions

1. $f(x) = x^2, g(x) = 2x - 1$

$$f \circ g(x) = f(g(x)) = f(2x - 1) = (2x - 1)^2 = 4x^2 - 4x + 1 = F$$

2. $f(x) = x^2, g(x) = 2x - 1$

$$g \circ f(x) = g(f(x)) = g(x^2) = 2(x^2) - 1 = 2x^2 - 1 = E$$

3. $g(x) = 2x - 1, h(x) = x - 5$

$$g \circ h(x) = g(h(x)) = g(x - 5) = 2(x - 5) - 1 = 2x - 10 - 1 = 2x - 11 = A$$

4. $g(x) = 2x - 1, h(x) = x - 5$

$$h \circ g(x) = h(g(x)) = h(2x - 1) = (2x - 1) - 5 = 2x - 6 = C$$

5. $f(x) = x^2, h(x) = x - 5$

$$f \circ h(x) = f(h(x)) = f(x - 5) = (x - 5)^2 = x^2 - 10x + 25 = B$$

6. $f(x) = x^2, h(x) = x - 5$

$$h \circ f(x) = h(f(x)) = h(x^2) = x^2 - 5 = D$$

13. The implied domain of $f(x) = 3x - 4$ is the set of x for which $f(x)$ is a valid function. As there is no way for us to divide by zero we can plug any number in for x . Therefore the answer is \mathbb{R}

15. Likewise for $h(x) = 15x - 3$ there is no way to divide by zero and any value for x that we plug in makes sense for the function, so the implied domain is \mathbb{R}

17. For $g(x) = \frac{2x-4}{x+3}$ if $x=-3$ then we divide by zero. This however is the only value which messes up the function so we get that the implied domain is $\mathbb{R} - \{-3\}$

18. For the function $h(x) = \frac{3x^2-4x+5}{-3x+4}$ if $x = \frac{4}{3}$, then we divide by zero. This is the only value for x for which the function doesn't make sense so the implied domain is $\mathbb{R} - \{\frac{4}{3}\}$

2 Other Problems

1. Summing across the n th row:

$$n = 0 : Sum = 1 = 2^0$$

$$n = 1 : Sum = 2 = 2^1$$

$$n = 2 : Sum = 4 = 2^2$$

$$n = 3 : Sum = 8 = 2^3$$

$$n = 4 : Sum = 16 = 2^4$$

$$n = 5 : Sum = 32 = 2^5$$

$$n = 6 : Sum = 64 = 2^6$$

So the sum across the n th row is 2^n

2. f is a constant function, as regardless of the input of \clubsuit , $f(\clubsuit) = \pi$.
Since the domain of f is \mathbb{N} we have that the object \clubsuit must be a natural number.
3. As smiley face does not exist in LaTeX let us redefine the function g as $g(\spadesuit) = \frac{\spadesuit}{(\spadesuit-3)}$. Then the implied domain of g is $\mathbb{R} - \{3\}$ as 3 is the only number which doesn't make sense as an input. The range of g is $\mathbb{R} - \{1\}$ as we discussed in class.