Math 1050-006 Homework 1 Solutions

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1 Book Problems

1.1 Sets and Numbers

1. True
2. False, $4 \notin \{14, 44, 43, 24\}$
3. False, $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$, $\frac{1}{3} \notin \mathbb{Z}$
4. False, $\mathbb{N} = \{1, 2, 3, 4, ...\}$, $-5 \notin \mathbb{N}$
5. True
6. True
13. False, $-2 \notin \mathbb{N}, therefore \{-2, 3, 0\} \not\subseteq \mathbb{N}$
14. True
15. True
16. False, $\sqrt{2} \notin \mathbb{Q}, therefore \left\{\sqrt{2}, 271\right\} \not\subseteq \mathbb{Q}$

1.2 Rules for Numbers

1. True
2. False, $3(x + y) = 3x + 3y \neq 3x + y$
6. False, $2 \in [2, 5]$, but $2 \notin (2, 5]$
7. True
11. True
12. False, $-17 \in [-17, \infty)$, but $-17 \notin (-17, \infty)$, therefore $[-17, \infty) \not\subseteq (-17, \infty)$.
16. False, $10 \notin (2, 8)$, therefore $\{3, 10, 7\} \not\subseteq (2, 8)$
17. True

1.3 Functions

1. $f(5) = \frac{1}{5^2} = \frac{1}{25}$
2. $f(10) = \frac{1}{10^2} = \frac{1}{100}$
3. $g(1) = \frac{2(1)^2-(1)+1}{(1)^2+1} = \frac{2-1+1}{1+1} = 1$
4. $g(-3) = \frac{2(-3)^2-(-3)+1}{(-3)^2+1} = \frac{18+3+1}{9+1} = \frac{22}{10} = 2.2$
5. $h(0) = 14$ (constant function h(x)=14)
6. $h(\frac{\pi^2}{6}) = 14$ (constant fucntion h(x)=14)
7. $id(15) = 15$ (identity function)
8. $id(-4) = -4$ (identity function)

1.4 Sequences

1. Neither, difference between 2 and 7 is 5, but the difference between 7 and 14 is 7, therefore not an arithmetic sequence. Also we need to multiply 2 by $\frac{7}{2}$ to get 7, but we need to multiply 7 by 2 to get 14, so not a geometric sequence.
2. Arithmetic, add 4 to each term to get the next term.
3. Geometric, multiply each term by -1 to get the next term

7. $a_1 = -1, d = 5$
8. $a_1 = 2, d = -12$

13. $a_1 = -5, r = 5$
14. $a_1 = 4, r = -2$

16. 5,7,9,11,... is an arithmetic sequence with $a_1 = 5, d = 2$, using the prediction equation for arithmetic sequences ($a_n = a_1 + (n-1)d$) we get that
$a_{4223} = a_1 + (4223-1)(d) = 5 + (4222)(2) = 5 + 8444 = 8449$
17. 4,1,-2,-5,... is an arithmetic sequence with $a_1 = 4, d = -3$ Therefore
$a_{5224} = a_1 + (5224-1)d = 4 + (5223)(-3) = 4 - 15669 = -15665$

18. 54,18,6,2,... is a geometric sequence with $a_1 = 54, r = \frac{1}{3}$, using the prediction equation for geometric sequences ($a_n = r^{(n-1)}a_1$) we get that
$a_7 = r^{7-1}a_1 = (\frac{1}{3})^6(54) = \frac{54}{3^6} = \frac{2*3^3}{3^6} = \frac{2}{3^3} = \frac{2}{27}$

19. -11,22,-44,88,... is a geometric sequence with $a_1 = -11, r = -2$.
Therefore $a_6 = (r)^{(6-1)}a_1 = (-2)^5(-11) = (-32)(-11) = 352$

23. $c_n = (3-n)(n+2)$, therefore if we let n=8 we get,
$c_8 = (3-8)(8+2) = (-5)(10) = -50$

1.5 Sums and Series

2. 3+3+3+....+3+3 (50 times)=3(50)=150

4. (-2)+(-2)+(-2)+(-2)+....+(-2)+(-2) (78 times)=(-2)(78)=-156

5. 1+2+3+4+5+...+38+39+40 is the finite sum of the first 40 terms of an arithmetic sequence which has equation $\frac{n}{2}(a_1 + a_n)$. Here
$a_1 = 1, n = 40, a_n = 40$. Therefore
$\frac{n}{2}(a_1 + a_n) = \frac{40}{2}(1+40) = (20)(41) = 820$

8. (2(1)-1)+(2(2)-1)+(2(3)-1)+(2(4)-1)+(2(5)-1)=1+3+5+7+9=25

13. Sum of the first 80 terms of the sequence 53,54,55,56,57,.... First note that this sequence is an arithmetic sequence with $a_1 = 53, d = 1$. We can therefore use the arithmetic sequence sum equation
($\frac{n}{2}(a_1 + a_n)$). However first we need to solve for a_n which we can do using the arithmetic sequence prediction equation ($a_n = a_1 + (n-1)d$). As n=80, we have $a_{80} = a_1 + (80-1)d = 53 + (79)1 = 132$ Therefore
$\frac{n}{2}(a_1 + a_n) = \frac{80}{2}(53 + 132) = (40)(185) = 7400$

16. Sum all of the terms in the geometric sequence $7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, ...$ This is a geometric sequence with $a_1 = 7, r = \frac{2}{3}$, Therefore we can use the sum of a geometric sequence formula, ($\frac{a_1}{(1-r)}$) Which for this sequence we get: $\frac{7}{1-\frac{2}{3}} = \frac{7}{\frac{1}{3}} = (7)(3) = 21$

17. Sum all of the terms in the geometric sequence $25, 15, 9, \frac{27}{5}, ...$ This is a geometric sequence with $a_1 = 25, r = \frac{3}{5}$.
Therefore $\frac{a_1}{(1-r)} = \frac{25}{1-\frac{3}{5}} = \frac{25}{\frac{2}{5}} = (25)(\frac{5}{2}) = \frac{125}{2}$

2 Other Problems

1. The difference between an infinite set and a sequence is that order matters in a sequence, but not in an infinite set.

2. g(x)=x is not an identity function. To be an identity function the Domain and the Target need to be the same, but for g(x), the Domain=$\mathbb{N}$ and the Target=$\mathbb{R}$. As $\mathbb{N} \neq \mathbb{R}$ we have that g(x) is not an identity function.

3. A sequence can be both an arithmetic and a geometric sequence, for example consider the sequence 1,1,1,1,1,1,.... which is an arithmetic sequence with $a_1 = 1, d = 0$ and a geometric sequence with $a_1 = 1, r = 1$

4. The fibonacci sequence 1,1,2,3,5,8,13,21,... is neither an arithmetic sequence nor a geometric sequence. Notice that the difference subsequent terms in the sequence is 0,1,1,2,3,5,8,... is not constant, therefore not arithmetic. Furthermore we would have to multiply 1 by 2 to get 2, but multiply 2 by $\frac{3}{2}$ to get, so r is not constant and therefore the fibonacci sequence is also not geometric.