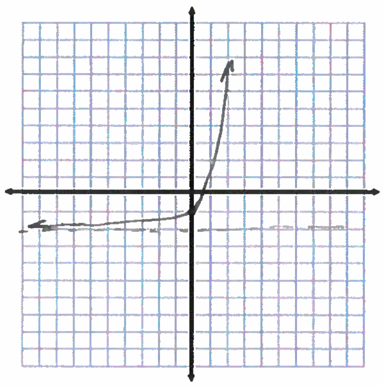
Solutions

Midterm 3
Practice Test

Graph1-8. State: Domain, Range, x-intercept(s), and y intercept

1) f(x)= $e^x - 2$ shift down 2



x int(s) $x = \log_e(2)$

y int. $y = -1$

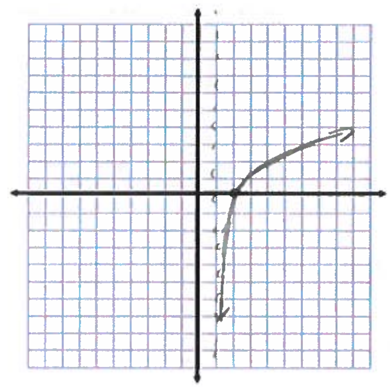
D= $\mathbb{R}$ R= $(-2, \infty)$

2) g(x)= $\ln(x-1)$ shift → 1

$\ln(x-1) = 0$

$x - 1 = e^0 = 1$

$x = 2$



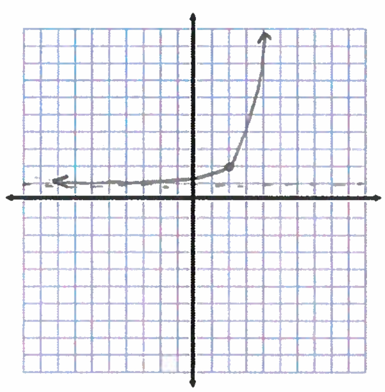
x int(s) $x = 2$

y int. none

D= $(1, \infty)$ R= $\mathbb{R}$

3) f(x)= $e^{x-2} + 1$

right 2 up 1



x int(s) none

y int. $y = e^{-2} + 1$

D= $\mathbb{R}$ R= $(1, \infty)$

4) m(x)= $\ln(x+1) - 2$

left 1 down 2

$\ln(x+1) - 2 = 0$

$e^{\ln(x+1)} = e^2$

$x + 1 = e^2$

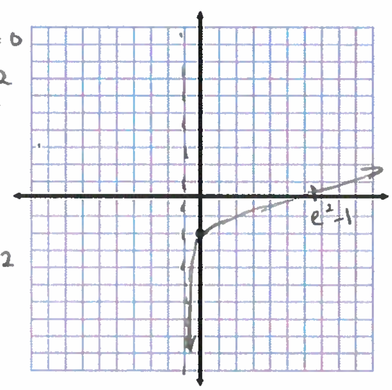
$x = e^2 - 1$

$\ln(0+1) - 2$

$\ln(1) - 2$

$= 0 - 2$

$y = -2$



x int(s) $x = e^2 - 1$

y int. $y = -2$

D= $(-1, \infty)$ R= $\mathbb{R}$

$-\ln(x-3)+1=0$

$e^{\ln(x-3)} = e^{1}$

$x = 3+e$

① left 2

5) $f(x) = -e^{x+2}+3$

② flip over x-axis ③ up 3

$-e^{x+2}+3=0$

$-e^{x+2}=-3$

$e^{x+2}=3$

$x+2=\ln(3)$

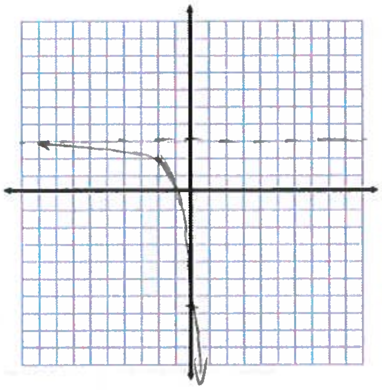
$x=\ln(3)-2$

$-e^{0+2}+3$

$=-e^{2}+3$

$=3-e^{2}$

≈ -7



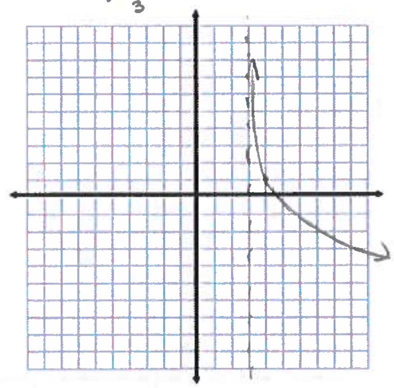
x int(s) $x=\ln(3)-2$

y int. $y=3-e^{2}$

D= $\mathbb{R}$ R= $(-\infty, 3)$

6) $g(x) = -\ln(x-3)+1$

② flip over x ① right 3 ③ up 1



x int(s) $x=3+e$

y int. none

D= $(3, \infty)$ R= $\mathbb{R}$

7) $f(x) = e^{2-x}+1 = e^{-(x-2)}+1$

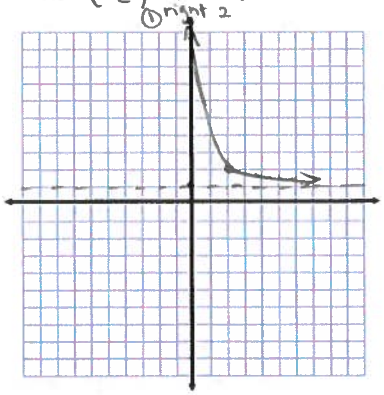
$= \left(\frac{1}{e}\right)^{x-2}+1$

① right 2 ② up 1

transform $\left(\frac{1}{e}\right)^{x}$ (decay)

$e^{2-0}+1$

$=e^{2}+1$



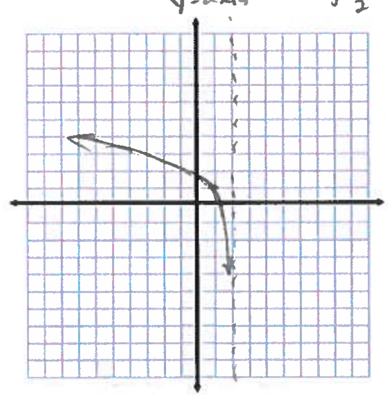
x int(s) none

y int. $y=e^{2}+1$

D= $\mathbb{R}$ R= $(1, \infty)$

8) $m(x) = \ln(2-x)+1 = \ln(-(x-2))+1$

① flip over y-axis ② right 2 ③ up 1



x int(s) $x=2-\frac{1}{e}$

y int. $y=\ln(2)+1$

D= $(-\infty, 2)$ R= $\mathbb{R}$

$\ln(2-0)+1 = \ln(2)+1$

$\ln(2-x)+1=0$

$e^{\ln(2-x)} = e^{-1}$

$2-x=e^{-1}=\frac{1}{e}$

$x=2-\frac{1}{e}$

Simplify each:

9) $\log_3 \frac{1}{81} = \log_3\left(\frac{1}{3^4}\right) = \log_3\left(3^{-4}\right) = \boxed{-4}$

10) $\log_2 \sqrt[3]{4} = \log_2\left(4^{1/3}\right) = \log_2\left(\left(2^2\right)^{1/3}\right) = \log_2\left(2^{2/3}\right) = \boxed{2/3}$

11) $\log_{16} \frac{1}{8} = \frac{\log_2\left(\frac{1}{8}\right)}{\log_2(16)} = \frac{\log_2\left(2^{-3}\right)}{\log_2\left(2^4\right)} = \boxed{\frac{-3}{4}}$

12) $\log_4 1 = \boxed{0}$ b/c $\log_a(1) = 0$ for all $a > 0,\ a \neq 1$

13) $\log_{10} 10{,}000 = \log_{10}\left(10^4\right) = \boxed{4}$

14) $\log_e e^3 = \log_e\left(e^3\right) = 3 \log_e(e) = 3 \cdot 1 = \boxed{3}$

15) $\log_{\frac{1}{2}} 32 = \frac{\log_2(32)}{\log_2\left(\frac{1}{2}\right)} = \frac{5}{-1} = \boxed{-5}$

Approximate each; state the integers between which each expression lays.

16) $\log_2 20$

exponent	1	2	3	4	5
	2	4	8	16 *	32

$\Rightarrow \boxed{4 < \log_2(20) < 5}$

17) $\log_3 100$

exponent	1	2	3	4	5
	3	9	27	81 *	243

$\boxed{4 < \log_3(100) < 5}$

17.5) $\log_{(1/3)} 30$

exponent	-4	-3	-2	-1	0	1	2
	81 *	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

$\boxed{-4 < \log_{\frac{1}{3}}(30) < -5}$

Solve each:

18) $\log_2 x - \log_2(x-1) = 3$

$$\Rightarrow \log_2\left(\frac{x}{x-1}\right) = 3 \Rightarrow \cancel{2}^{\cancel{\log_2}\left(\frac{x}{x-1}\right)} = 2^3$$

$$\frac{x}{x-1} = 8 \Rightarrow x = 8(x-1)$$

$$x = 8x - 8$$

$$7x = 8 \qquad \boxed{x = 8/7}$$

19) $4e^{x-2} - 7 = 2$

$$4e^{x-2} = 9$$

$$e^{x-2} = 9/4 \Rightarrow \ln(e^{x-2}) = \ln(9/4)$$

$$x - 2 = \ln(9/4)$$

$$\boxed{x = 2 + \ln(9/4)}$$

20) $(2^{2x+3}) = 5(2^{x-2})$

$$\frac{2^{2x+3}}{2^{x-2}} = 5 \Rightarrow 2^{(2x+3)-(x-2)} = 5 \qquad \log_2(2^{x+5}) = \log_2(5)$$

$$2^{x+5} = 5 \Rightarrow x + 5 = \log_2(5)$$

$$\boxed{x = \log_2(5) - 5}$$

21) $e^{2x} = \dfrac{e^3}{e^{x-1}}$

$$e^{2x}e^{x-1} = e^3 \Rightarrow \frac{e^{2x}e^{x-1}}{e^3} = 1 \Rightarrow e^{2x+(x-1)-3} = 1$$

$$e^{3x-4} = 1 \Rightarrow \ln(e^{3x-4}) = \ln(1)$$

$$3x - 4 = 0$$

$$\boxed{x = 4/3}$$

22) $\log_6(x-1) + \log_6(x+2) = \log_6(x^2+2)$

$$6^{\log_6((x-1)(x+2))} = 6^{\log_6(x^2+2)}$$

$$(x-1)(x+2) = x^2 + 2 \Rightarrow \cancel{x^2} + 2x - x - 2 = \cancel{x^2} + 2$$

$$x - 2 = 2$$

$$\boxed{x = 4}$$

23) $\log_2(x-2) + \log_2(x+1) = 3$

$$\log_2((x-2)(x+1)) = 3$$

$$\Rightarrow 2^{\log_2((x-2)(x+1))} = 2^3 \Rightarrow (x-2)(x+1) = 8$$

$$x^2 - 2x + x - 2 = 8$$

$$x^2 - x - 10 = 0$$

$$x = \frac{1 \pm \sqrt{1+40}}{2}$$

$$x = \frac{1 \pm \sqrt{41}}{2}$$

$$\boxed{x = \frac{1+\sqrt{41}}{2}}$$

24) $3(2^{x+2}) = 2^{3x-1}$

$$3 = \frac{2^{3x-1}}{2^{x+2}} = 2^{(3x-1)-(x+2)}$$

$$\Rightarrow 3 = 2^{2x-3} \Rightarrow \log_2(3) = \log_2(2^{(2x-3)})$$

$$\log_2(3) = 2x - 3 \Rightarrow \boxed{x = \frac{3 + \log_2(3)}{2}}$$

25) $12 = 3e^{x+2} + 5$

$$3e^{x+2} = 7$$

$$e^{x+2} = 7/3 \qquad \ln(e^{x+2}) = \ln(7/3)$$

$$x + 2 = \ln(7/3) \Rightarrow \boxed{x = \ln(7/3) - 2}$$

26) $\log_{10}(x-1)^{-2} = -4$

~~27) 33 24 48~~

$$\log_{10}\left[(x-1)^{-2}\right] = -4$$

$$-2\log_{10}(x-1) = -4 \rightarrow 10^{\log_{10}(x-1)} = 10^2$$

$$x - 1 = 100 \qquad \boxed{x = 101}$$

27) $|3x-4|<8$

$\Rightarrow \quad 3x-4<8$ and/or $-(3x-4)<8$

$3x<12 \qquad 3x-4>-8$

$x<4 \qquad 3x>-4$

$x>-4/3$

$\Rightarrow \boxed{-4/3<x<4}$

28) $|2x+3|-5\geq 2$

$|2x+3|\geq 7 \Rightarrow \quad 2x+3\geq 7$ and/or $-(2x+3)\geq 7$

$2x\geq 4 \qquad 2x+3\leq -7$

$x\geq 2 \qquad 2x\leq -10$

$x\leq -5$

$\Rightarrow \boxed{x\geq 2 \text{ or } x\leq -5}$

29) $|3x+1|-5<-2$

$|3x+1|<3 \Rightarrow \quad 3x+1<3$ and/or $-(3x+1)<3$

$3x<2 \qquad 3x+1>-3$

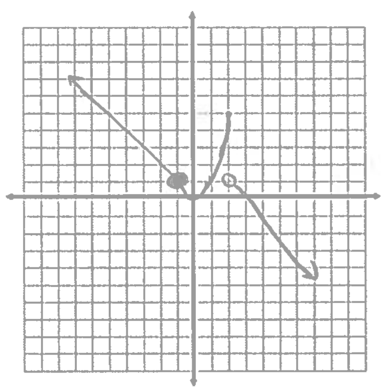
$x<2/3 \qquad 3x>-4$

$x>-4/3$

$\Rightarrow \boxed{-4/3<x<2/3}$

30) Graph

$$f(x)=\begin{cases} |x| & x\in(-\infty,-1) \\ x^2 & x\in[-1,2] \\ 3-x & x\in(2,\infty) \end{cases}$$



For f(x) above, find:

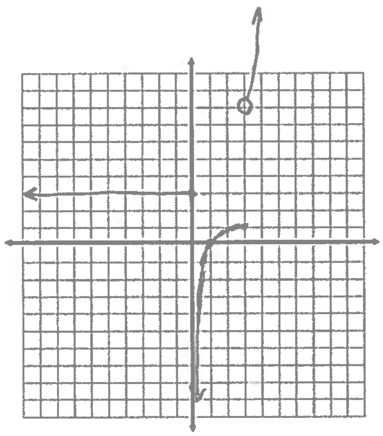
f(-2)=__2__ $\qquad -2\in(-\infty,-1) \Rightarrow f(-2)=|-2|=2$

f(0)=__0__ $\qquad 0\in[-1,2] \Rightarrow f(0)=0^2=0$

f(2)=__4__ $\qquad 2\in[-1,2] \Rightarrow f(2)=2^2=4$

31) Graph

$$f(x)=\begin{cases} 3 & x\in(-\infty,0] \\ \log_3 x & x\in(0,3] \\ 2^x & x\in(3,\infty) \end{cases}$$



find:

f(-1)= 3 $\quad -1\in(-\infty,0] \Rightarrow f(-1)=3$

f(0)= 3 $\quad 0\in(-\infty,0] \Rightarrow f(0)=3$

f(3)= 1 $\quad 3\in(0,3] \Rightarrow f(3)=\log_3(3)=1$

Solve the following systems

32) elimination

$x+y=5$

$+\ x-y=3$

$2x=8 \Rightarrow \boxed{x=4}$

$4+y=5$

$\boxed{y=1}$

33) substitution

$2x-3y=12$

$y=x-10$

$2x-3(x-10)=12$

$2x-3x+30=12$

$-x+30=12$

$-x=-18$

$\boxed{x=18}$

$y=18-10$

$\boxed{y=8}$

34) substitution

$8x+7y=38$

$3x-5y=-1$

$3x-5y=-1$

$3x=5y-1 \Rightarrow$

$x=\frac{5}{3}y-\frac{1}{3}$

$8\left(\frac{5}{3}y-\frac{1}{3}\right)+7y=38$

$\frac{40}{3}y-\frac{8}{3}+7y=38$

$\frac{61}{3}y-\frac{8}{3}=\frac{114}{3}$

$\frac{61}{3}y=\frac{122}{3}$

$\boxed{y=2}$

$3x-5(2)=-1$

$3x-10=-1$

$3x=9$

$\boxed{x=3}$

~~35)~~

~~$3x+2y-z=4$~~

~~$2x-3x+z=-1$~~

~~$x+y+z=6$~~

* 35 on last page

Given the following matrices:

$A=\begin{pmatrix} 1 & 2 \end{pmatrix}$ $B=\begin{pmatrix} -2 & 0 & 1 \end{pmatrix}$ $C=\begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ $D=\begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$

$F=\begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$ $G=\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

36) AB

$$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \end{bmatrix} = \boxed{\text{undefined}}$$

$1 \times 2 \quad 1 \times 3$ $\quad 2 \neq 1$ inner dimensions don't match

37) 3A + B

$$3 * \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 1 \end{bmatrix} = \boxed{\text{undefined}}$$

vectors aren't same length

38) AC

$$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot 2 & 1\cdot -1 + 2\cdot 0 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & -1 \end{bmatrix}}$$

39) DF

$$\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1\cdot -1 + 0\cdot 2 & -1\cdot 0 + 0\cdot 1 \\ 2\cdot -1 + 1\cdot 2 & 2\cdot 0 + 1\cdot 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

40) CG

$$\begin{bmatrix} 1 & -1 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + -1\cdot 2 \\ 2\cdot 0 + 0\cdot 2 \end{bmatrix} = \boxed{\begin{bmatrix} -2 \\ 0 \end{bmatrix}}$$

41) D – C

$$\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -1-1 & 0-(-1) \\ 2-2 & 1-0 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}}$$

42) C – D

$$\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1-(-1) & -1-0 \\ 2-2 & 0-1 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix}}$$

43) 4C

$$4\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 4 & -4 \\ 8 & 0 \end{bmatrix}}$$

35) $3x + 2y - z = 4$

$* \ 2x - 3x + z = -1 \Rightarrow -x + z = -1$

$x + y + z = 6$

$z = x - 1$

$3x + 2y - (x-1) = 4$

$x + y + (x-1) = 6$

$\Rightarrow$

$2x + 2y = 3$

$- \ 2x + y = 7$

$\boxed{y = -4}$

$\Rightarrow 2x + (-4) = 7$

$2x = 11$

$\boxed{x = 11/2}$

$z = x - 1$

$z = \frac{11}{2} - 1 = \boxed{\frac{9}{2} = z}$