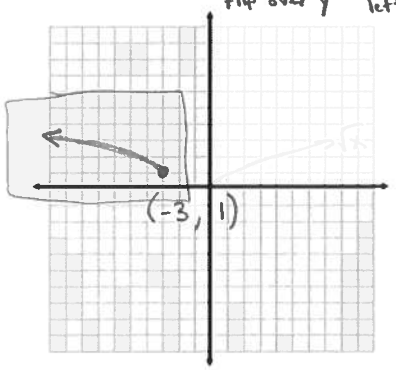
Midterm 2
Practice Test

Graph1 – 6. State: Domain, Range, x-intercept(s), and y intercept

1) f(x)= $\sqrt[2]{3-x}+1 = \sqrt{-x+3} + 1$

flip over y left 3 up 1



x int(s) none

y int. none

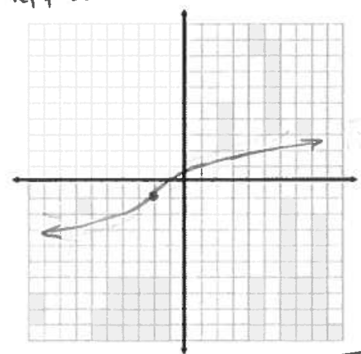
D= $(-\infty, -3]$ R= $[1, \infty)$

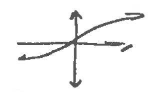
Where is f(x) > 0?

2) g(x)= $\sqrt[3]{x+2}-1$

left 2 down 1

start with $\sqrt[3]{x}$





$\sqrt[3]{x+2} - 1 = 0$
$\sqrt[3]{x+2} = 1$
$x+2 = 1$
$x = -1$

x int(s) $x = -1$

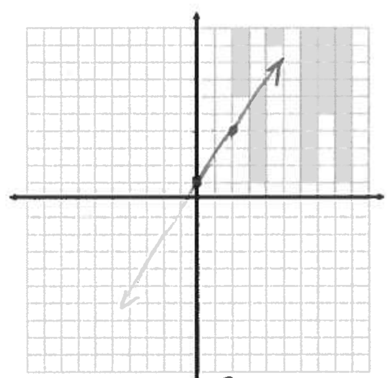
y int. $y = \sqrt[3]{2} - 1$

D= $\mathbb{R}$ R= $\mathbb{R}$

Where is g(x) > 0?

3) $y=\frac{3}{2}x+1$

Slope = $\frac{3}{2}$
y-int = 1



$\frac{3}{2}x + 1 = 0 \Rightarrow \frac{3}{2}x = -1$

$x = -\frac{2}{3}$

x int(s) $x = -\frac{2}{3}$

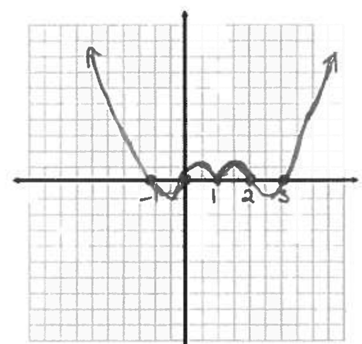
y int. $y = 1$

D= $\mathbb{R}$ R= $\mathbb{R}$

4) $m(x) = -2x(x-3)^3(x+1)(2-x)(x-1)^2$
$= 2x(x-3)^3(x+1)(x-2)(x-1)^2$

Roots
$x=0$ single
$x=3$ triple
$x=-1$ single
$x=2$ single
$x=1$ double

y-int = 0



$x=0,\ x=3$
$x=-1,\ x=2$

x int(s) $x = 1$ y int. $y = 0$

Leading term $2x^8$

D= $\mathbb{R}$ R=NO

Where is m(x) > 0? $\boxed{x<1,\ 0<x<2,\ x>3}$

$LT = 2(x)(x)^3(x)(x)(x)^2$
$= 2x^8$

down 1 ④

$-\sqrt{x+2}+1=0$

$-\sqrt{x+2}=-1$

$\sqrt{x+2}=1$

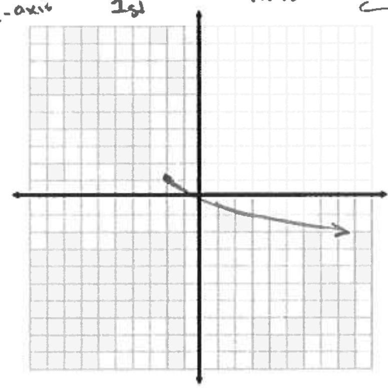
$x+2=1$

$x=-1$

$-\sqrt{2}+1$

5) $f(x)= -\sqrt[2]{x+2}+1$

flip over x-axis second; left 2 1st; up 1 third; Start with $\sqrt{x}$



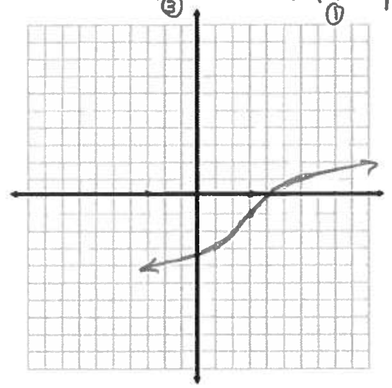
x int(s) $x=-1$

y int. $y=-\sqrt{2}+1$

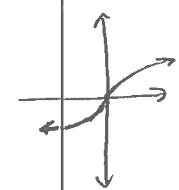
D= $[-2,\infty)$ R= $(-\infty,1]$

6) $g(x)= -\sqrt[3]{3-x}-1$ $= -\sqrt[3]{-x+3}-1$

flip over x ③; flip over y ①; left 3 ②



Start with $\sqrt[3]{x}$



$-\sqrt[3]{3-x}-1=0$

$\sqrt[3]{3-x}=-1$

$3-x=-1$

$x=4$

x int(s) $x=4$

y int. $y=-1-\sqrt[3]{3}$

D= $\mathbb{R}$ R= $\mathbb{R}$

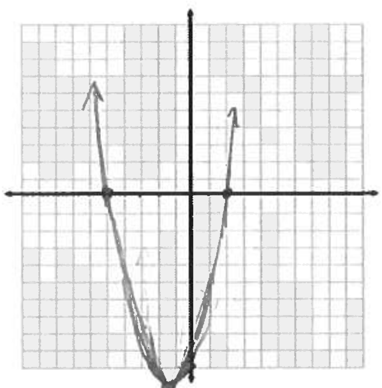
Graph 7 – 9. State: Domain, Range, x-intercept(s), and y intercept.

7) $h(x) = x^2 + 3x - 10$

Complete the Square $= \left(x+\frac{3}{2}\right)^2 - \frac{49}{4}$

$x = \frac{-3 \pm \sqrt{49}}{2}$

$= \frac{-3 \pm 7}{2} = 2, -5$



vertex $\left(-\frac{3}{2}, -\frac{49}{4}\right)$

x int(s) $x=2,\ x=-5$

y int. $y=-10$

D= $\mathbb{R}$ R= $\left[-\frac{49}{4},\infty\right)$

Where is $h(x) > 0$? $x>2,\ x<-5$

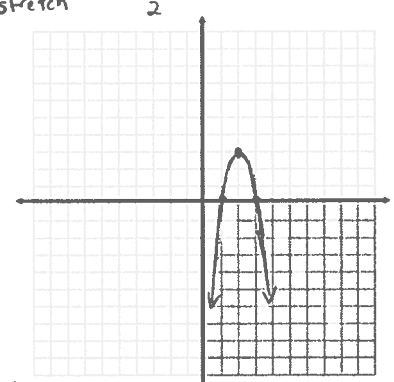
8) $n(x) = -3(x-2)^2 + 3$

flip + stretch over x-axis; right 2; up 3

$-3(-2)^2+3$

$=-12+3$

$=-9$



$-3(x-2)^2+3=0$

$(x-2)^2=1 \Rightarrow x = 2\pm\sqrt{1}$

$=1, 3$

x int(s) $x=1,\ x=3$

y int. $y=-9$

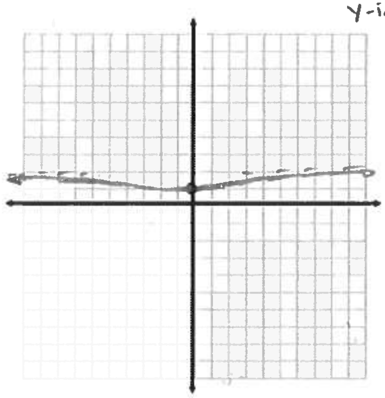
D= $\mathbb{R}$ R= $(-\infty,3]$

Where is $n(x) > 0$? $1<x<3$

Graph 9-11. State the x-int., y-int, vertical asymptotes (VA), and leading term (LT)

9) $f(x)=\dfrac{2x^2+1}{x^2+1}$

Top Roots (TR) = none
Bottom Roots (BR) = none
Leading Term (LT) = 2
y-int = 1

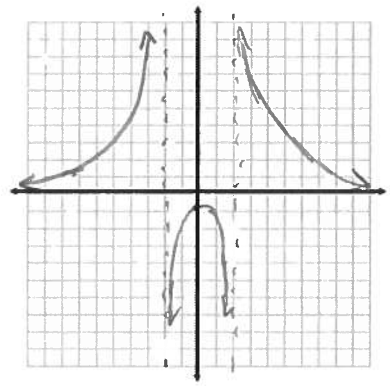


x int __none__ y int __y = 1__

VA= __none__ LT= __2__

10) $f(x)=\dfrac{3}{x^2-4}$

TR = none
BR = x = 2, x = −2
LT = $\frac{3}{x^2}$
y-int = $\frac{-3}{4}$



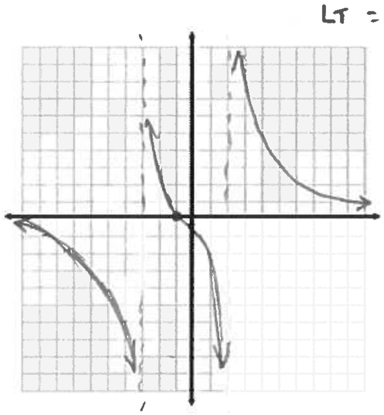
x int __none__ y int __y = $\frac{-3}{4}$__

VA= __x = 2, x = −2__ LT= __$\frac{3}{x^2}$__

11) $f(x)=\dfrac{x+1}{x^2+x-6}$

x^2+x-6
$=(x+3)(x-2)$

TR = −1
BR = −3, 2
single
LT = $\frac{x}{x^2}=\frac{1}{x}$
y-int = $\frac{-1}{6}$

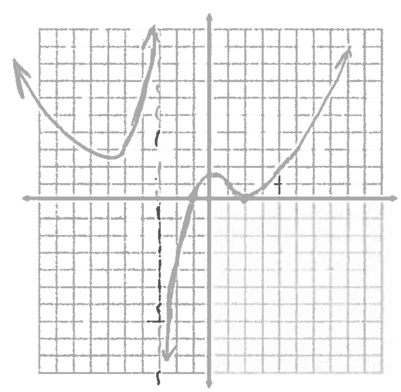


x int __x = −1__ y int __y = $\frac{-1}{6}$__

VA= __x = −3, x = 2__ LT= __$\frac{1}{x}$__

12) $f(x)=\dfrac{(x+1)(x-2)^2}{x+3}$

TR x = −1, 2 (double)
BR x = −3
LT = $\frac{x^3}{x}=x^2$
y-int = $\frac{4}{3}$



x int __x = −1, x = 2__ y int __y = $\frac{4}{3}$__

VA= __x = −3__ LT= __x^2__

13) Simplify: $\dfrac{12x^3-13x^2+9x-2}{x^2-1}$

$$= \boxed{12x - 13 + \frac{21x-15}{x^2-1}}$$

$$\begin{array}{r} 12x - 13 \\ x^2-1 \overline{)\,12x^3 - 13x^2 + 9x - 2} \\ -(12x^3 + 0x^2 - 12x) \\ \hline -13x^2 + 21x - 2 \\ -(-13x^2 + 0x + 13) \\ \hline 21x - 15 \end{array} = \text{Remainder}$$

14) Is $(x-1)$ a factor of $x^5+3x^4+x^3-x^2-x-1$?

if $(x-1)$ is a factor then $x=1$ is a root

Explain: $\Rightarrow (1)^5 + 3(1)^4 + (1)^3 - (1)^2 - (1) - 1$

$= 1+3+1-1-1-1 = 2 \neq 0$

$\Rightarrow (x-1)$ is not a factor

15) COMPLETING THE SQUARE of $f(x) = 3x^2 + 6x - 5$ by.

$a = 3$
$b = 6$
$c = -5$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 3\left(x + \frac{6}{2\cdot 3}\right)^2 + (-5) - \frac{(6)^2}{4\cdot 3}$$

$$= 3(x+1)^2 - 5 - \frac{36}{12} = \boxed{3(x+1)^2 - 8}$$

Using the completed square form find the roots of f(x):

$f(x) = 0 \Rightarrow 3(x+1)^2 - 8 = 0$

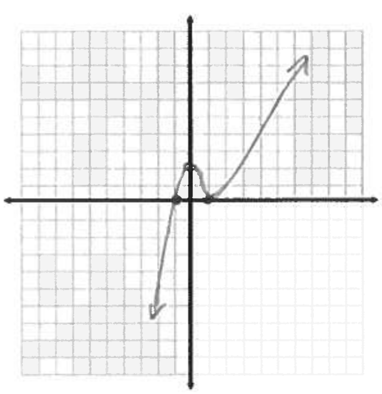
$3(x+1)^2 = 8$

$(x+1)^2 = \frac{8}{3}$

$x+1 = \pm\sqrt{\frac{8}{3}}$

$$\boxed{x = -1 \pm 2\sqrt{\tfrac{2}{3}}}$$

16) Factor $f(x)=3x^3-4x^2-x+2$ completely. Hint x = 1 is a root.
Sketch the graph, show x and y intercepts



if $x = 1$ is a root $(x-1)$ is a factor

$$\begin{array}{r} 3x^2 - x - 2 \\ x-1 \overline{)\,3x^3 - 4x^2 - x + 2} \\ -(3x^3 - 3x^2) \\ \hline -x^2 - x \\ -(-x^2 + x) \\ \hline -2x + 2 \end{array}$$

$f(x) = (x-1)(3x^2 - x - 2)$

$= 3(x-1)(x-1)\left(x + \frac{2}{3}\right) = 3(x-1)^2\left(x + \frac{2}{3}\right)$

Discriminant $= (-1)^2 - 4(3)(-2)$

$= 1 + 24 = 25$

$$x = \frac{-(-1) \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6} = 1, \frac{-2}{3}$$

$\Rightarrow$ factors $= (x-1), \left(x + \frac{2}{3}\right)$

double

x-int $= x = 1$, $x = \frac{-2}{3}$

y-int : $y = 2$

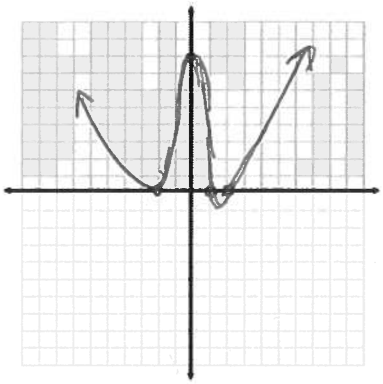
$LT = 3x^3$

17) Factor $f(x)=x^4+x^3-6x^2-4x+8$ completely. Hint x = 1 and x = -2 are roots. Sketch the graph. Show x and y intercepts.

$x=1 \Rightarrow (x-1)$ is a factor

$$\begin{array}{r} x^3+2x^2-4x-8 \\ x-1 \overline{\big) x^4+x^3-6x^2-4x+8} \\ -(x^4-x^3) \\ \hline 2x^3-6x^2 \\ -(2x^3-2x^2) \\ \hline -4x^2-4x \\ -(-4x^2+4x) \\ \hline -8x+8 \end{array}$$

$\Rightarrow f(x)=(x-1)(x^3+2x^2-4x-8)$



$x=-2 \Rightarrow (x+2)$ is a factor

$$\begin{array}{r} x^2-4 \\ x+2 \overline{\big) x^3+2x^2-4x-8} \\ -(x^3+2x^2) \\ \hline 0x^2-4x-8 \\ -(-4x-8) \\ \hline 0 \end{array}$$

$\Rightarrow f(x)=(x-1)(x+2)(x^2-4)$

$x^2-4=(x+2)(x-2)$

$\Rightarrow f(x)=(x-1)(x-2)(x+2)^2$

x-ints = roots: $x=1$, $x=2$, $x=-2$ (double)

y-int = 8

$LT = x^4$

18) Find all real roots for $f(x)=x^3-2x^2-2x+1$ Hint (x + 1) is a factor.

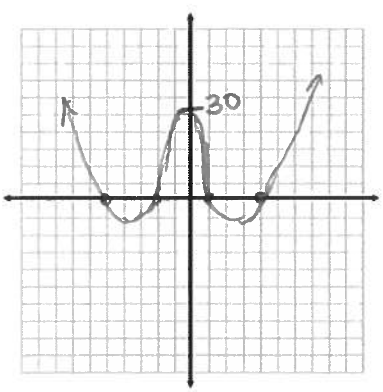
$$\begin{array}{r} x^2-3x+1 \\ x+1 \overline{\big) x^3-2x^2-2x+1} \\ -(x^3+x^2) \\ \hline -3x^2-2x \\ -(-3x^2-3x) \\ \hline x+1 \end{array}$$

Discriminant $= (-3)^2-4(1)(1) = 9-4=5$

$x=\dfrac{3\pm\sqrt{5}}{2} \Rightarrow$ Roots: $x=-1$, $x=\frac{3}{2}+\frac{\sqrt{5}}{2}$, $x=\frac{3}{2}-\frac{\sqrt{5}}{2}$

19) Find all real roots for $f(x)=x^4+3x^3-15x^2-19x+30$ Hint (x + 2) and (x – 1) are factors. Sketch the graph, show x and y intercepts.

$$\begin{array}{r} x^3+4x^2-11x-30 \\ x-1 \overline{\big) x^4+3x^3-15x^2-19x+30} \\ -(x^4-x^3) \\ \hline 4x^3-15x^2 \\ -(4x^3-4x^2) \\ \hline -11x^2-19x \\ -(-11x^2+11x) \\ \hline -30x+30 \end{array}$$



$(x-1)(x^3+4x^2-11x-30)$

(x+2) factor $\Rightarrow$

$$\begin{array}{r} x^2+2x-15 \\ x+2 \overline{\big) x^3+4x^2-11x-30} \\ -(x^3+2x^2) \\ \hline 2x^2-11x \\ -(2x^2+4x) \\ \hline -15x-30 \end{array}$$

Disriminant $4-4(1)(-15) = 64$

$x=\dfrac{-2\pm\sqrt{64}}{2}=-1\pm 4$

$x=-5$, $x=3$

x-ints = $x=1$, $x=-2$, $x=-5$, $x=3$

y-int = 30

$LT = x^4$

$\Rightarrow (x-1)(x+2)(x^2+2x-15)=(x-1)(x+2)(x+5)(x-3)$

Simplify:

20) $\dfrac{\frac{2x^2}{x}-\frac{3}{x}}{\frac{5}{x}} = \dfrac{\frac{2x^2-3}{x}}{\frac{5}{x}} = \dfrac{2x^2-3}{x}\cdot\dfrac{x}{5} = \dfrac{2x^2-3}{5}$

$= \boxed{\frac{2}{5}x^2 - \frac{3}{5}}$

21) $\dfrac{5x^4}{\left(\frac{3x^3}{2}\right)\left(\frac{2x}{5}\right)} = \dfrac{\frac{5x^4}{1}}{\frac{3x^3\cdot 2x}{2\cdot 5}} = \dfrac{\frac{5x^4}{1}}{\frac{6x^4}{10}} = \dfrac{5x^4}{1}\cdot\dfrac{10}{6x^4} = \dfrac{50}{6}$

$= \boxed{\frac{25}{3}}$

22) $\dfrac{2}{x}+\dfrac{x}{3} = \dfrac{2}{x}\cdot\dfrac{3}{3} + \dfrac{x}{3}\cdot\dfrac{x}{x}$

$= \dfrac{6}{3x} + \dfrac{x^2}{3x} = \boxed{\frac{x^2+6}{3x}}$

23) $\dfrac{x+3}{2x}+\dfrac{4}{x-1} = \dfrac{(x+3)}{2x}\cdot\dfrac{(x-1)}{(x-1)} + \dfrac{4}{(x-1)}\cdot\dfrac{2x}{2x} = \dfrac{(x+3)(x-1)}{2x(x-1)} + \dfrac{8x}{2x(x-1)}$

$= \dfrac{x^2+3x-x-3+8x}{2x(x-1)} = \boxed{\frac{x^2+10x-3}{2x(x-1)}}$

Find the value of x

24) $\dfrac{3x-1}{x+2}=4$

$3x-1 = 4(x+2)$

$3x-1 = 4x+8$

$3x-4x = 8+1$

$-x = 9$

$\boxed{x=-9}$

25) $\dfrac{x^2+4}{x^2-x-2}=1$

$x^2+4 = 1(x^2-x-2)$

$x^2+4 = x^2-x-2$

$x^2-x^2+x = -2-4$

$\boxed{x=-6}$

26) Solve for x: $\sqrt[2]{3x-2}-6=-4$

$\sqrt{3x-2} = -4+6$

$\sqrt{3x-2} = 2$

square both sides $\quad 3x-2 = (2)^2$

$3x = 4+2$

$3x = 6$

$\boxed{x=2}$

27) Solve the inequality for x: $10 \le 2-x^3$

$2-x^3 \ge 10$

$-x^3 \ge 8$

$x^3 \le -8$

$x \le \sqrt[3]{-8}$

$\boxed{x \le -2}$

sign didn't change when taking cube root ⇒ inequality doesn't change direction

28) Solve the inequality for x: $\sqrt{2x-1} \ge 4$

$\left(\sqrt{2x-1}\right)^2 \ge (4)^2$

$2x-1 \ge 16$

$2x \ge 17$

$\boxed{x \ge \frac{17}{2}}$

29) Solve the inequality for x: $\frac{2}{x-3} \le 5$

$\frac{2}{x-3} \le 5$

when we multiply by $x-3$ it could be either +/-

if $x>3$, then $2 \le 5(x-3)$

$2 \le 5x-15$

$17 \le 5x$

$x \ge \frac{17}{5}$

if $x<3$, then $2 \ge 5(x-3)$

$2 \ge 5x-15$

$17 \ge 5x$

$x \le \frac{17}{5}$

⇒ if $x>3$, $x \ge \frac{17}{5}$ ⇒ $x \ge \frac{17}{5}$

if $x<3$, $x \le \frac{17}{5}$ ⇒ $x<3$

30) Solve the inequality for x: $\frac{3}{2-x} \le 4$

$\frac{3}{2-x} \le 4$

if $x<2$, then $3 \le 4(2-x)$

$3 \le 8-4x$

$-5 \le -4x$

$\frac{5}{4} \ge x$

$x<2,\ x \le \frac{5}{4}$

⇒ $\boxed{x \le \frac{5}{4}}$

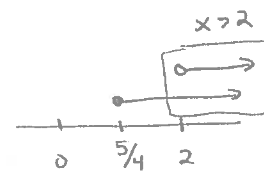
if $x>2$, then $3 \ge 4(2-x)$

$3 \ge 8-4x$

$-5 \ge -4x$

$\frac{5}{4} \le x$

$x>2,\ x \ge \frac{5}{4}$ ⇒ $\boxed{x>2}$



31) Solve the inequality for x: $\frac{1}{5-x} \ge 3$

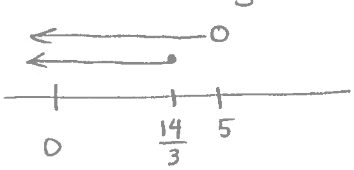
if $x<5$, then $1 \ge 3(5-x)$

$1 \ge 15-3x$

$-14 \ge -3x$

$\frac{14}{3} \le x$

⇒ $x<5$ + $\frac{14}{3} \le x$



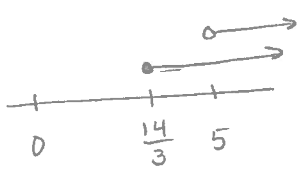
⇒ $\boxed{x \le \frac{14}{3}}$

if $x>5$, then $1 \le 3(5-x)$

$1 \le 15-3x$

$-14 \le -3x$

$\frac{14}{3} \ge x$



⇒ $\boxed{x>5}$