Solutions

Midterm 1: REVIEW

For #1-4, state if the sequence is arithmetic, geometric or neither.

1) 3, 30, 300, 3000,... $\boxed{\text{geometric}}$, $r = 10$

2) 1, 4, 9, 16,... $\boxed{\text{neither}}$, $a_i = i^2$

3) 27, 21, 15, 9,... $\boxed{\text{arithmetic}}$, $d = -6$

4) 40, 10, 10/4, 10/16,... $\boxed{\text{geometric}}$, $r = \frac{1}{4}$

For # 5-17, find the indicated sum:

5) If an = $3n^2 - 1$, find a_8

$a_8 = 3(8)^2 - 1 = 3(64) - 1 = 192 - 1 = \boxed{191}$

6) If an = $3 - n^2$, find a_{10}

$a_{10} = 3 - (10)^2 = 3 - 100 = \boxed{-97}$

7) What is the 27th term in the sequence -10, -7, -4, -1...?

arithmetic, $d = -3$ $\quad a_{27} = a_1 + (27-1)d = -10 + (26)(-3) = \boxed{-88}$

8) What is the 23rd term in the sequence 48, 24, 12, 6,...?

geometric, $r = \frac{1}{2}$ $\quad a_{23} = r^{23-1} a_1 = \left(\frac{1}{2}\right)^{22} 48 = \frac{2^4 \cdot 3}{2^{22}} = \boxed{\frac{3}{2^{18}}}$

9) $\sum_{i=1}^{12} 4 = \underbrace{4 + 4 + \cdots + 4}_{12 \text{ times}} = 12 \cdot 4 = \boxed{48}$

10) $\sum_{i=1}^{15} -2i + 1 = -2\sum_{i=1}^{15} i + \sum_{i=1}^{15} 1 = -2 \times \frac{15}{2}(1 + 15) + 15 \cdot 1$

arithmetic

$= -15 \cdot 16 + 15 = -15 \cdot 15 = -15^2 = \boxed{-225}$

11) $\sum_{i=1}^{\infty} 4\left(\frac{2}{3}\right)^{i-1} =$ geometric

$a_1 = 4\left(\frac{2}{3}\right)^0 = 4$

$r = \frac{2}{3}$, $\quad -1 < \frac{2}{3} < 1$ ✓

$S = \frac{a_1}{1-r} = \frac{4}{1 - \frac{2}{3}} = \frac{4}{1/3} = \boxed{12}$

12) $\sum_{i=1}^{\infty} \left(\frac{3}{4}\right)(4)^{i-1} =$ geometric

$a_1 = \frac{3}{4}(4)^0 = \frac{3}{4}$

$r = 4$

$-1 < 4 < 1$ X $\Rightarrow \boxed{\text{diverges}}$

13) $\sum_{i=1}^{\infty}\frac{4}{3^{i-1}} = \sum_{i=1}^{\infty} 4\left(\frac{1}{3}\right)^{i-1}$

geometric

$a_1 = 4\left(\frac{1}{3}\right)^0 = 4$

$r = \frac{1}{3}$

$-1 < \frac{1}{3} < 1$ ✓

$\Rightarrow S = \frac{a_1}{1-r} = \frac{4}{1-\frac{1}{3}} = \frac{4}{2/3} = \boxed{6}$

14) What is the sum of the first 83 terms for 91+86+81+76+…?

Arithmetic, $a_1 = 91$, $d = -5$

$a_{83} = a_1 + (83-1)d = 91 + 82(-5) = -319$

$\sum_{i=1}^{83} a_i = \frac{83}{2}(91 + -319) = 83\cdot(-114) = \boxed{-9462}$

15) Find the sum of the following infinite series: 81+27+9+3+…?

Geometric, $a_1 = 81$, $r = \frac{1}{3}$

$-1 < \frac{1}{3} < 1$ ✓

$S = \frac{81}{1-\frac{1}{3}} = \frac{81}{2/3} = \boxed{\frac{243}{2}}$

16) Find the sum of the following infinite series: 25+15+9+27/5+…

Geometric, $a_1 = 25$, $r = \frac{3}{5}$

$-1 < \frac{3}{5} < 1$ ✓

$S = \frac{25}{1-3/5} = \frac{25}{2/5} = \boxed{\frac{125}{2}}$

17) Find the sum of the following infinite series: ½+1+2+4+…

Geometric, $a_1 = \frac{1}{2}$, $r = 2$

$-1 < 2 \not< 1$ X $\Rightarrow$ $\boxed{\text{diverges}}$

Answer each of the following:

18) You're on vacation and have brought with you 3 pair of shorts, 4 shirts and 2 pair of shoes. How many different outfits can you make?

$\underset{\text{Shorts}}{\underline{3}} \times \underset{\text{Shirts}}{\underline{4}} \times \underset{\text{Shoes}}{\underline{2}} = \boxed{24}$

19) There are 18 jelly-bellies in a jar, each are different flavors. How many different ways can you select 4 jelly-bellies?

Order does not matter $\Rightarrow$ $\binom{18}{4} = \frac{18!}{(18-4)!\,4!} = \frac{18\cdot17\cdot16\cdot15\cdot\cancel{14!}}{\cancel{14!}\cdot 4!} = \boxed{\frac{18\cdot17\cdot16\cdot15}{4\cdot3\cdot2}}$

20) There are 100 sales people at a large corporation. The president of the corporation has decided she will select a head of sales, assistant to the head of sales, and secretary from the pool of 100. How many different options does the president have?

$\underset{\text{HoS}}{\underline{100}} \times \underset{\text{AHoS}}{\underline{99}} \times \underset{\text{Secretary}}{\underline{98}} = \boxed{100\cdot99\cdot98}$

21) If you have four different gifts and you want to give each one of your four best friends one of them, how many different ways can you give your gifts?

Ordering 4 objects $= \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = \boxed{4!}$

22) You work for a large corporation that likes to give gifts to politicians. There is a pool of 12 different gifts that you can give and you have to select a specific gift for three different people, how many ways could you give the gifts?

$\boxed{12 \times 11 \times 10} = \boxed{\frac{12!}{9!}}$

23) You just got your first credit card and need to choose a four-digit pin number.

a) Assuming you must use 0-9 for each digit but there are no other restrictions, how many possible pin number are there?

allows repetition $10 \times 10 \times 10 \times 10 = \boxed{10^4}$

b) You've decided that you want a pin number where the four digits are unique (no repeated digits). How many possible pin numbers are there if you make this restriction?

$10 \times 9 \times 8 \times 7 = \boxed{\frac{10!}{6!}}$

24) There are 10 people interested in learning how to use a new interactive technology, but the instructor can only train 3 people. How many ways might she choose the three people?

order does not matter $= \binom{10}{3} = \frac{10!}{7!\,3!} = \boxed{\frac{10\cdot 9\cdot 8}{3\cdot 2}}$

25) You're at the gym and need to work out your arms, abs and legs. There are 3 machines for arms, two for abs and 6 for legs. You only have time to use one machine for each area (arms, abs and legs). How many different ways could you select the machines you will use?

$\underset{\text{arms}}{3} \times \underset{\text{abs}}{2} \times \underset{\text{legs}}{6} = \boxed{36}$

26) What is $\binom{12}{5}$?

$\frac{12!}{(12-5)!\,5!} = \frac{12!}{7!\,5!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}\,5!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \boxed{11\cdot 9\cdot 8}$

27) Write $(x-3)^4$ in expanded notation using the binomial theorem

$= \sum_{i=0}^{4} \binom{4}{i} x^i (-3)^{4-i} = \binom{4}{0}(-3)^4 + \binom{4}{1}x(-3)^3 + \binom{4}{2}x^2(-3)^2 + \binom{4}{3}x^3(-3) + \binom{4}{4}x^4$

$= \boxed{81 - 27\cdot 4x + 6\cdot 9x^2 - 4\cdot 3x^3 + x^4}$

28) If $f(x) = 2x - 3$ and $g(x) = x^2 - 3$, then what is

a) $f \circ g(x)$ $= f(g(x)) = f(x^2-3) = 2(x^2-3) - 3 = 2x^2 - 6 - 3 = \boxed{2x^2 - 9}$

b) $g \circ f(x)$ $= g(f(x)) = g(2x-3) = (2x-3)^2 - 3$

$= 4x^2 - 12x + 9 - 3 = \boxed{4x^2 - 12x + 6}$

For 29 – 32, state the implied domain

29) $f(x)=x^2-3x-10$ $\boxed{\mathbb{R}}$

30) g(x)= $\sqrt{5-2x}$ $\boxed{\mathbb{R}-(\frac{5}{2},\infty)}$

even root, negative $\Rightarrow$ $5-2x<0$

$2x>5$ $x>5/2$

31) $h(x)=\dfrac{x^2+5x-14}{x+3}$ $\boxed{\mathbb{R}-\{-3\}}$

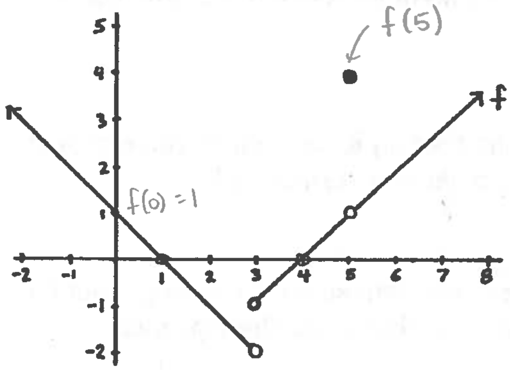
division by 0

$x+3=0$ $x=-3$

32) q(x)= $\sqrt[5]{4-3x}$

odd root $\Rightarrow$ no problem $\Rightarrow$ $\boxed{\mathbb{R}}$

Below is the graph of a function f(x). Use the graph to answer questions 26-30



33) What is f(0)? 33) 1

34) Find all x for f(x)=0. 34) $x=1$, $x=4$

35) What is f(5)? 35) 4

36) What is the domain of f(x)? ~~3~~ undefined at $x=3$ 36) $\mathbb{R}-\{3\}$

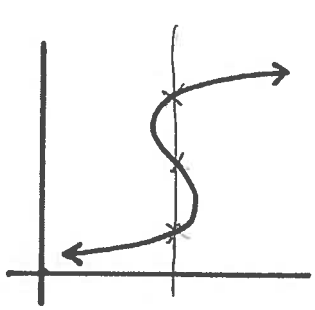
37) State the range of f(x) 37) $(-2,\infty)$

undefined for ≤ -2

38) State all x and y intercepts 38) x intercept(s) 1, 4

39) y intercept(s) 1

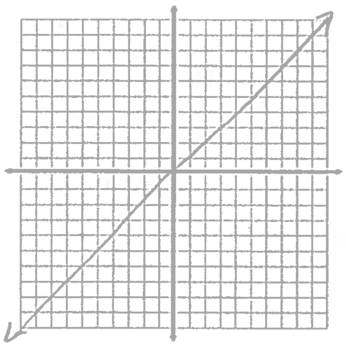
40) Is the picture below the graph of a function?



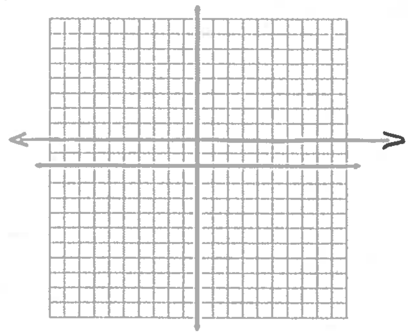
$\boxed{\text{no}}$, fails the vertical line test

For numbers 31 – 50 graph and state the domain and range of each.

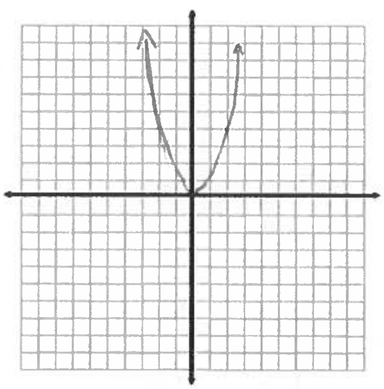
41) $f(x) = x$ D= $\mathbb{R}$ R= $\mathbb{R}$



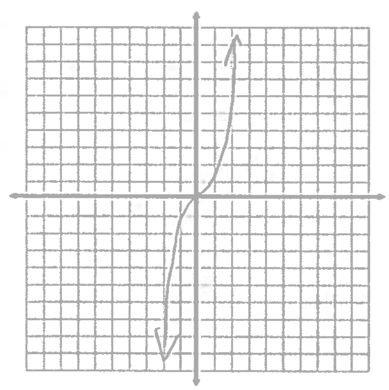
42) $f(x) = 2$ D= $\mathbb{R}$ R= 2



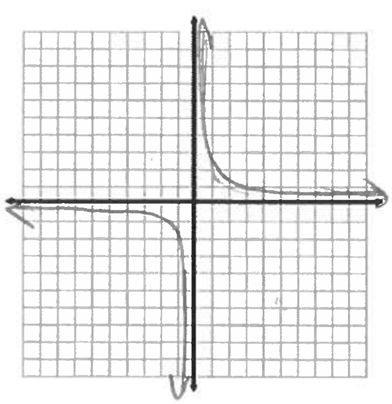
43) $f(x) = x^2$ D= $\mathbb{R}$ R= $[0, \infty)$



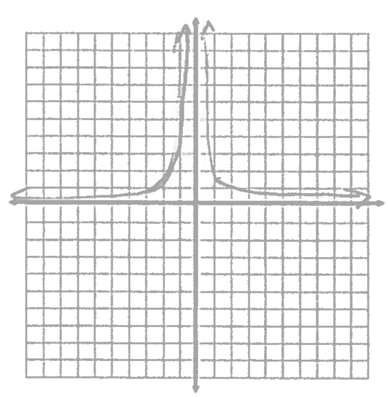
44) $f(x) = x^3$ D= $\mathbb{R}$ R= $\mathbb{R}$



45) $f(x) = 1/x$ D= $\mathbb{R} - \{0\}$ R= $\mathbb{R} - \{0\}$

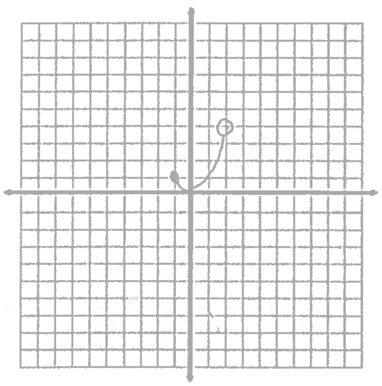


46) $f(x) = 1/x^2$ D= $\mathbb{R} - \{0\}$ R= $(0, \infty)$



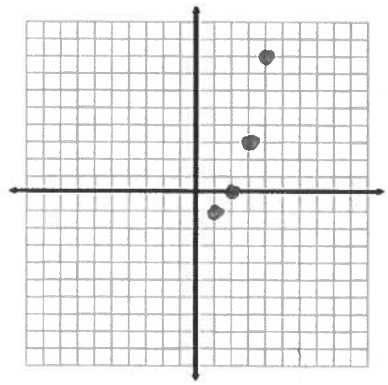
47) g : [-1,2) → ℜ where $g(x) = x^2$

D= [-1, 2) R= [0, 4)



48) h : {1,2,3,4} → ℜ where $h(x) = x^2 - 2x$

D= {1, 2, 3, 4} R= {-1, 0, 3, 8}



$h(1) = 1^2 - 2 \cdot 1$
$= -1$

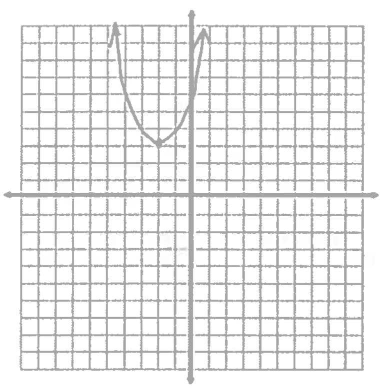
$h(2) = 2^2 - 2 \cdot 2$
$= 0$

$h(3) = 3^2 - 2 \cdot 3$
$= 3$

$h(4) = 4^2 - 2 \cdot 4$
$= 8$

49) $f(x) = (x - 2)^2 + 3$

D= ℝ R= [3, ∞)



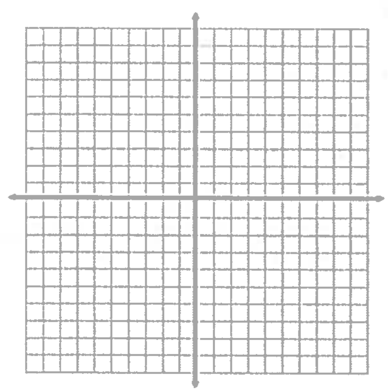
graph trans. of x^2

← 2 left

↑ 3 up

50) $f(x) = \sqrt{3 - x} + 1$

D= R=



Don't worry about this problem

48) If g(x) is an invertible function, and g(2)=7, then what is $g^{-1}(7)$?

$g^{-1}(7) = g^{-1}(g(2)) = \boxed{2}$

49) Find the inverse of f(x)= $\sqrt[3]{2x+4}$.

① $y = \sqrt[3]{2x+4}$ → ② $x = \sqrt[3]{2y+4}$ → ③ $x^3 = 2y+4$

swap x, y

$2y = x^3 - 4$

$y = \boxed{\frac{x^3-4}{2} = f^{-1}(x)}$

50) Find the inverse of g(x)= $\frac{x}{x+1}$

① $y = \frac{x}{x+1}$ → ② $x = \frac{y}{y+1}$ → ③ $x(y+1) = y$

$xy + x = y$

$xy - y = -x$

$y(x-1) = -x$

$y = \boxed{\frac{-x}{x-1} = g^{-1}(x)}$

51) Find the inverse of h(x) = 5 – 2x

① $y = 5-2x$

② $x = 5-2y$

$2y + x = 5$

$2y = 5 - x$

$y = \boxed{\frac{5-x}{2} = h^{-1}(x)}$