Final Review

Discrete Math

For #1-5 find the specific term:

1) Find the 18^{th} term for: $67, 61, 55, \ldots$

Arithmetic

$d = -6$

$a_1 = 67$

$a_{18} = a_1 + (18-1)d$

$= 67 + (17)(-6) = 67 - 102 = \boxed{-35}$

2) Find the 21^{st} term for: $-14, -10, -6, \ldots$

Arithmetic

$d = 4$

$a_1 = -14$

$a_{21} = a_1 + (21-1)d$

$= -14 + (20)(4) = -14 + 80 = \boxed{66}$

3) Find the 83^{rd} term for: $16, 8, 4, \ldots$

Geometric

$r = \frac{1}{2}$

$a_1 = 16$

$a_{83} = a_1 \cdot (r)^{(83-1)} = 16 \cdot \left(\frac{1}{2}\right)^{82} = \frac{16}{2^{82}}$

$= \frac{2^4}{2^{82}} = \boxed{2^{-78} = \left(\frac{1}{2}\right)^{78}}$

4) Find the 57^{th} term for: $-3, 6, -12, 24, \ldots$

Geometric

$r = -2$

$a_1 = 3$

$a_{57} = a_1 \cdot r^{56} = \boxed{3(-2)^{56}}$

5) Find the 32^{nd} term for: $\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots$

Geometric

$r = 2$

$a_1 = \frac{2}{3}$

$a_{32} = a_1 \cdot r^{31} = \left(\frac{2}{3}\right) \cdot 2^{31} = \boxed{\frac{2^{32}}{3^{31}}}$

For #6-12 find the given sum:

6) $\sum_{i=1}^{4}(i^2 - 1) = \underset{i=1}{\underline{1^2 - 1}} + \underset{i=2}{\underline{2^2 - 1}} + \underset{i=3}{\underline{3^2 - 1}} + \underset{i=4}{\underline{4^2 - 1}}$

$= 0 + 3 + 8 + 15 = \boxed{26}$

7) $\sum_{i=1}^{14}(2-5i) = \sum_{i=1}^{14} 2 - \sum_{i=1}^{14} 5i = \sum_{i=1}^{14} 2 - 5\sum_{i=1}^{14} i$ ← aritmetic

$= 14(2) - 5\left(\frac{14}{2}(1+14)\right)$

$= 28 - 5(7)(15)$

$= 28 - 525 = \boxed{-497}$

8) $\sum_{i=1}^{\infty}\left(\frac{4}{3^i}\right) = \sum_{i=1}^{\infty} 4\left(\frac{1}{3}\right)^i = \underset{i=1}{\frac{4}{3}} + \underset{i=2}{\frac{4}{9}} + \ldots$

Geometric

$r = \frac{1}{3}$

$a_1 = 4/3$

if $-1 < \frac{1}{3} < 1$ ✓ $\Rightarrow$ Sum $= \frac{a_1}{1-r} = \frac{4/3}{1-1/3}$

$= \frac{4/3}{2/3} = \boxed{2}$

9) $\sum_{i=1}^{\infty}\left(\frac{3}{2}\right)^{i-1} = \underset{i=1}{1} + \underset{i=2}{\frac{3}{2}} + \underset{i=3}{\frac{9}{4}}$

Geometric

$a_1 = 1$

$r = 3/2$

$3/2 > 1 \Rightarrow$ $\boxed{\text{diverges}}$

10) The sum of the first 18 terms of $-4 + -1 + 2 + \ldots$

Arithmetic

$a_1 = -4$

$d = 3$

$a_{18} = a_1 + (18-1)d$

$= -4 + 17 \cdot 3 = 47$

Sum $= \frac{18}{2}(a_1 + a_{18}) = \frac{18}{2}(-4 + 47) = 9 \cdot 43$

$= \boxed{387}$

11) The sum of the first 23 terms of $81 + 72 + 63 + \ldots$

Arithmetic

$a_1 = 81$

$d = -9$

$a_{23} = 81 + 22(-9) = -117$

Sum $= \frac{23}{2}(81 + -117)$

$= \frac{23}{2}(-36) = 23(-18)$

$= \boxed{-414}$

12) The infinite sum of $8 + 4 + 2 + ..$

Geometric

$a_1 = 8$

$r = \frac{1}{2}$

$-1 < \frac{1}{2} < 1 \Rightarrow$ Sum $= \frac{a_1}{1-r} = \frac{8}{1-1/2} = \frac{8}{1/2} = \boxed{16}$

For #13-21 find the following:

13) You're at Café Rio and want to order a burrito. There are three kinds of beans, two types of rice, and four kinds of meat fillings for the burrito. How many different kinds of burritos can you make if you select one of each? (You don't need to simplify)

Options Multiply

$$\underset{\text{meat}}{\underline{4}} \times \underset{\text{beans}}{\underline{3}} \times \underset{\text{rice}}{\underline{2}} = \boxed{24}$$

14) Write $\binom{9}{3}$ as an integer.

$$= \frac{9!}{(9-3)!\,3!} = \frac{9!}{6!\cdot 3!} = \frac{9\cdot 8\cdot 7\cdot \cancel{6!}}{\cancel{6!}\ 3!} = \frac{9\cdot 8\cdot 7}{3\cdot 2} = 3\cdot 4\cdot 7 = \boxed{84}$$

15) Suppose a set A contains 243 objects. How many 92 object subsets of A are there? (You don't need to simplify)

Want to choose 92 objects from a set of 243

order does not matter $= \binom{243}{92} = \boxed{\frac{243!}{151!\,92!}}$

16) How many ways are there to choose and order 49 objects from a collection of 304 objects? (You don't need to simplify)

Order 49 objects from 304

$$\Rightarrow \frac{n!}{(n-k)!} = \frac{304!}{(304-49)!} = \boxed{\frac{304!}{255!}}$$

17) How many different 4 letter words can you make with the letters MATH if you use each letter once? (You don't need to simplify)

Ordering 4 objects $\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 4! = \boxed{24}$

18) How many different ways can you order 21 different objects? (You don't need to simplify)

Ordering 21 objects $= \boxed{21!}$

19) There are 20 people competing for 3 scholarships each worth \$5000 per year. How many different ways can you select three different people? (You don't need to simplify)

Order doesn't matter

$$\binom{20}{3} = \frac{20!}{(20-3)!\,3!} = \boxed{\frac{20!}{17!\,3!}}$$

20) There are 20 people competing for a \$1000, \$3000, and \$5000 scholarship. How many different ways can you select the three different people? (You don't need to simplify)

Order <u>does</u> matter

$$\underline{20}\cdot\underline{19}\cdot\underline{18} = \boxed{\frac{20!}{17!}}$$

21) How many 3 people committees can you make from 12 people? (You don't need to simplify)

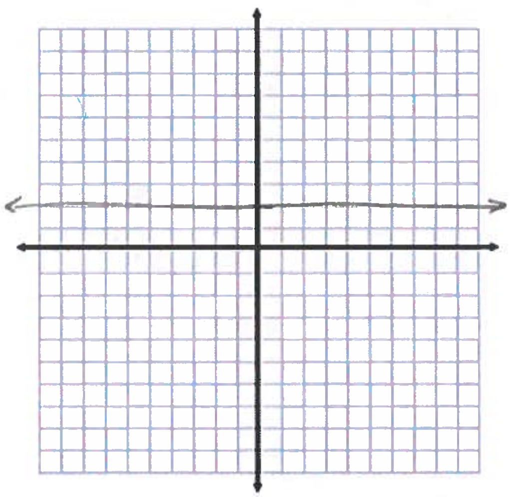
Order does <u>not</u> matter

$$\binom{12}{3} = \frac{12!}{(12-3)!\,3!} = \boxed{\frac{12!}{9!\,3!}}$$

Graphing

Graph and state the x-intercept(s), y-intercept, domain (D), range (R), leading term (LT), end behavior (EB), or vertical asymptote (VA) when specified:

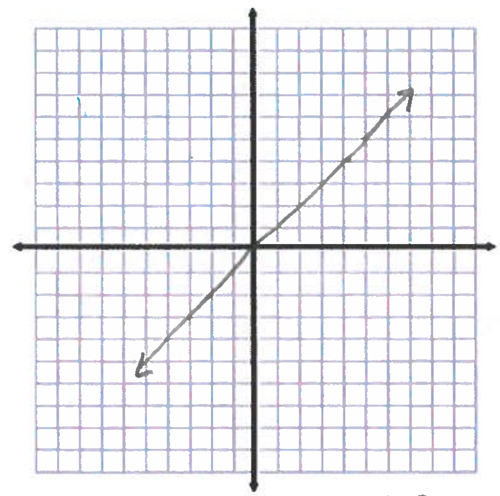
22) Graph $f(x) = 2$



x-int(s)= none y-int= $y = 2$

D= $\mathbb{R}$ R= $\{2\}$

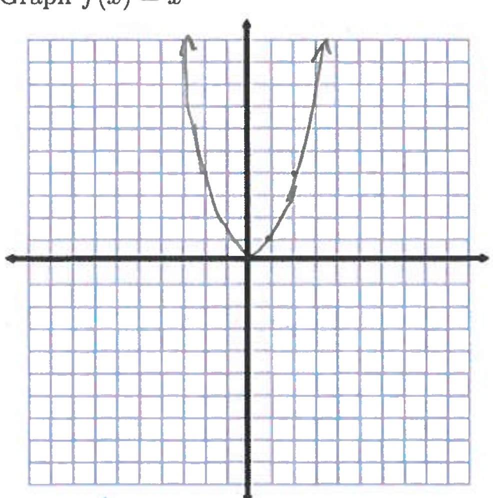
23) Graph $f(x) = x$



x-int(s)= $x=0$ y-int= $y=0$

D= $\mathbb{R}$ R= $\mathbb{R}$

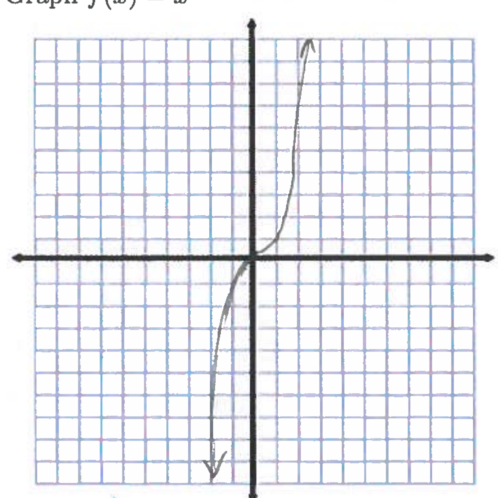
24) Graph $f(x) = x^2$



x-int(s)= $x=0$ y-int= $y=0$

D= $\mathbb{R}$ R= $[0, \infty)$

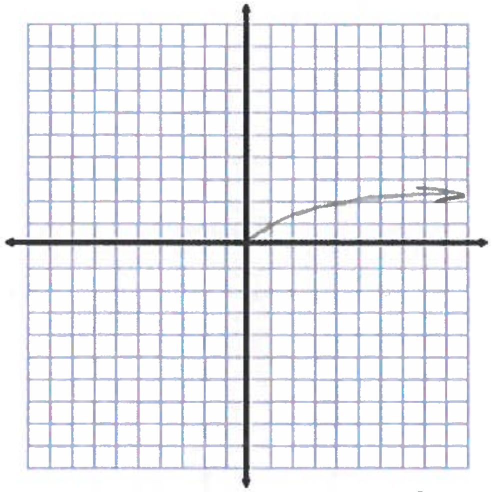
25) Graph $f(x) = x^3$



x-int(s)= $x=0$ y-int= $y=0$

D= $\mathbb{R}$ R= $\mathbb{R}$

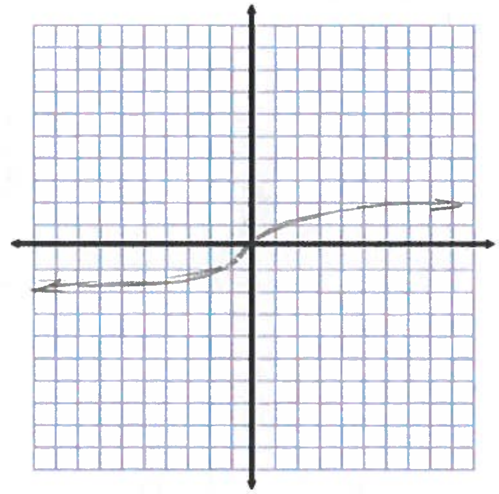
26) Graph $f(x) = \sqrt[2]{x}$ or any even root function



x-int(s)= $x = 0$ y-int= $y = 0$

D= $[0, \infty)$ R= $[0, \infty)$

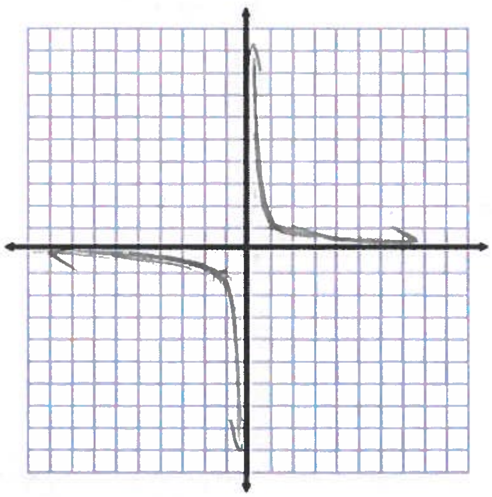
27) Graph $f(x) = \sqrt[3]{x}$ or any odd root function



x-int(s)= $x = 0$ y-int= $y = 0$

D= $\mathbb{R}$ R= $\mathbb{R}$

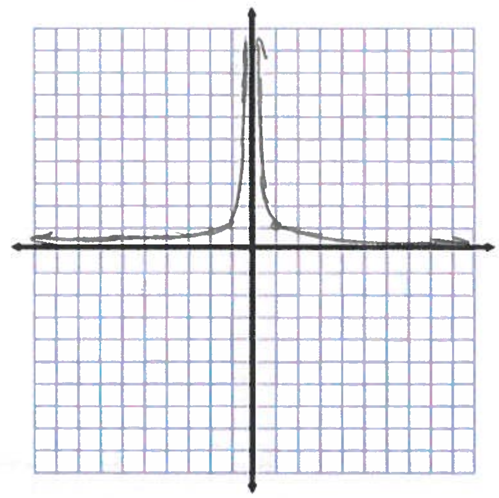
28) Graph $f(x) = \dfrac{1}{x}$



x-int(s)= none y-int= none

D= $\mathbb{R} - \{0\}$ R= $\mathbb{R} - \{0\}$

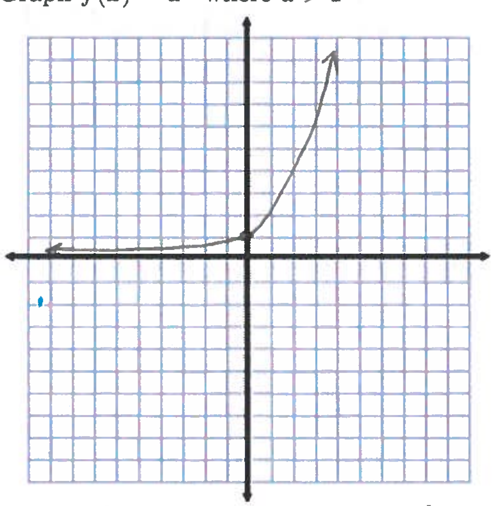
29) Graph $f(x) = \dfrac{1}{x^2}$



x-int(s)= none y-int= none

D= $\mathbb{R} - \{0\}$ R= $(0, \infty)$

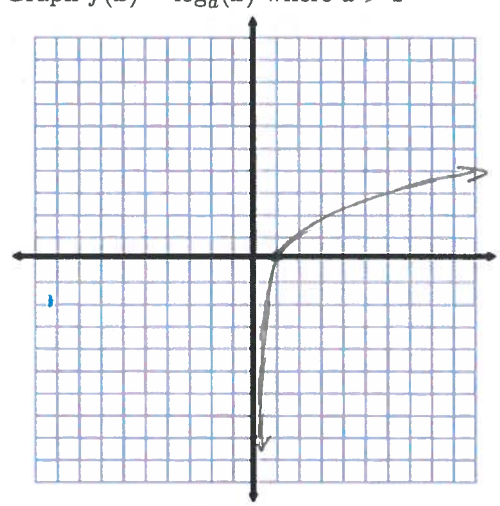
30) Graph $f(x) = a^x$ where $a > 1$



x-int(s)= none y-int= $y = 1$

D= $\mathbb{R}$ R= $(0, \infty)$

31) Graph $f(x) = \log_a(x)$ where $a > 1$

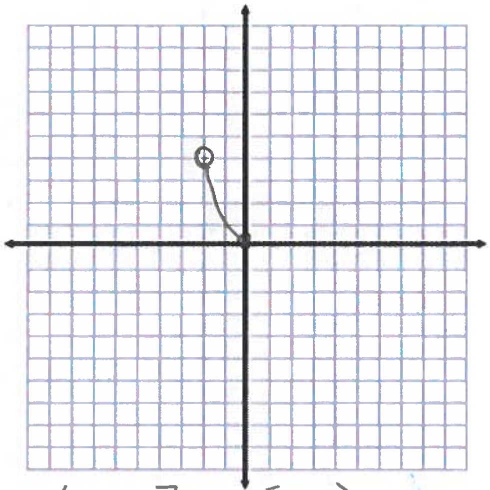


x-int(s)= $x = 1$ y-int= none

D= $(0, \infty)$ R= $\mathbb{R}$

Domain

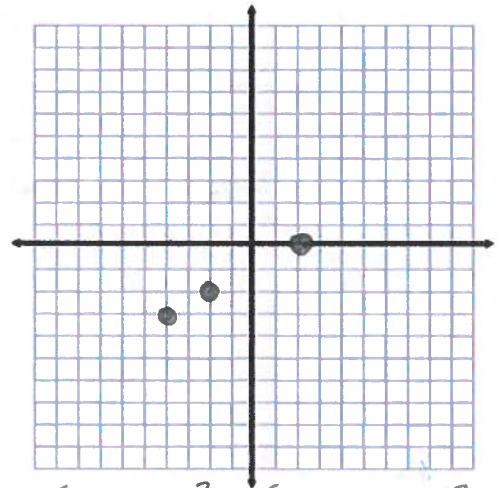
32) Graph $f : (-2, 0] \to \mathbb{R}$ where
$f(x) = x^2$



D= $(-2, 0]$ R= $[0, 4)$

Domain

33) Graph $g : \{-4, -2, 2\} \to \mathbb{R}$ where
$g(x) = \frac{1}{2}x - 1$



D= $\{-4, -2, 2\}$ R= $\{-3, -2, 0\}$

$g(-4) = \frac{1}{2}(-4) - 1 = -3$

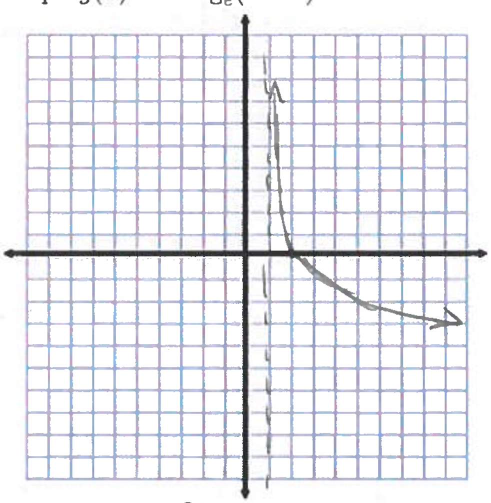
$g(-2) = \frac{1}{2}(-2) - 1 = -2$

$g(2) = \frac{1}{2}(2) - 1 = 0$

② flip over x-axis

right 1 ①

34) Graph $g(x) = -\log_e(x - 1)$

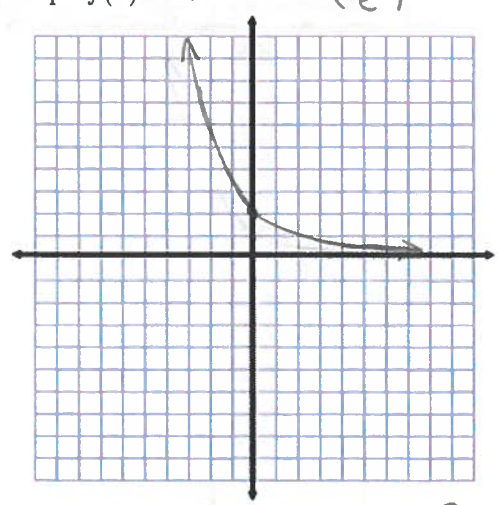


x-int(s)= $x = 2$ y-int= none
D= $(1, \infty)$ R= $\mathbb{R}$

$e > 1 \Rightarrow$ logarithmic growth

vertical stretch

35) Graph $f(x) = 2e^{-x} = 2\left(\frac{1}{e}\right)^x$

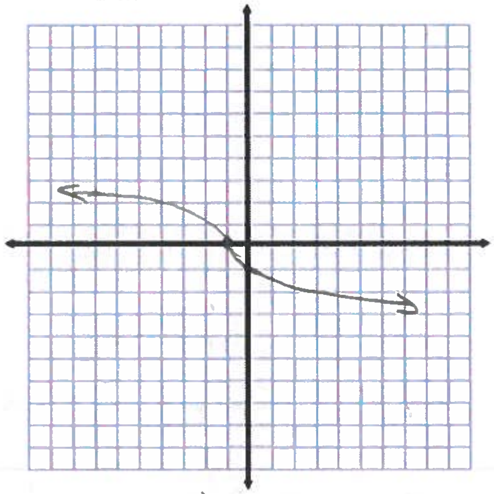


x-int(s)= none y-int= $y = 2$
D= $\mathbb{R}$ R= $(0, \infty)$

$0 < \frac{1}{e} < 1 \Rightarrow$ exponential decay

② flip over x-axis   left 1 ①

36) Graph $n(x) = -\sqrt[3]{x+1}$



x-int(s)= $x=-1$ y-int= $y=-1$

D= $\mathbb{R}$ R= $\mathbb{R}$

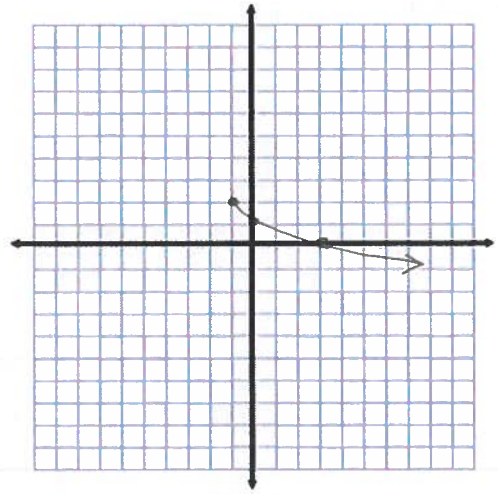
$n(x)=0=-\sqrt[3]{x+1} \Rightarrow x+1=0$

$x=-1$

$n(0)=-\sqrt[3]{0+1}=-\sqrt[3]{1}=-1$

② flip over x   ① left 1   up 2 ③

37) Graph $h(x) = -\sqrt{x+1}+2$



x-int(s)= $x=3$ y-int= $y=1$

D= $[-1,\infty)$ R= $(-\infty,2]$

$-\sqrt{x+1}+2=0$

$-\sqrt{x+1}=-2$

$\sqrt{x+1}=2$

$x+1=4$

$x=3$

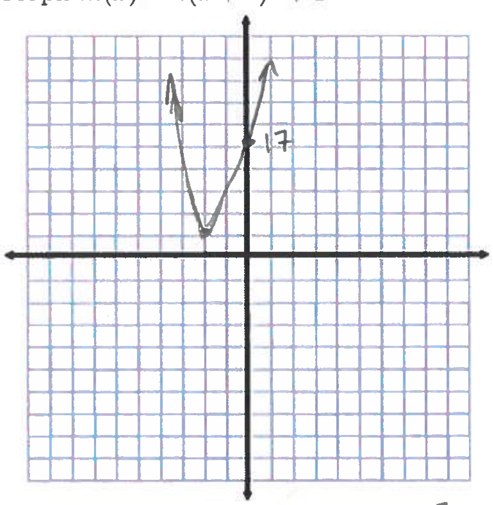
$h(0)=-\sqrt{0+1}+2$

$=-1+2$

$=1$

vertical stretch   left 2   up 1

38) Graph $m(x) = 4(x+2)^2+1$



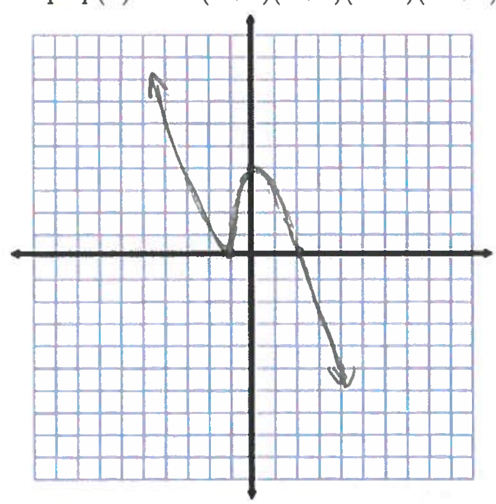
x-int(s)= none y-int= $y=17$

D= $\mathbb{R}$ R= $\mathbb{R}$

vertex= $(-2,1)$

$4(2)^2+1=17$

39) Graph $p(x) = -2(x+1)(x+1)(x-2)(x^2+1)$



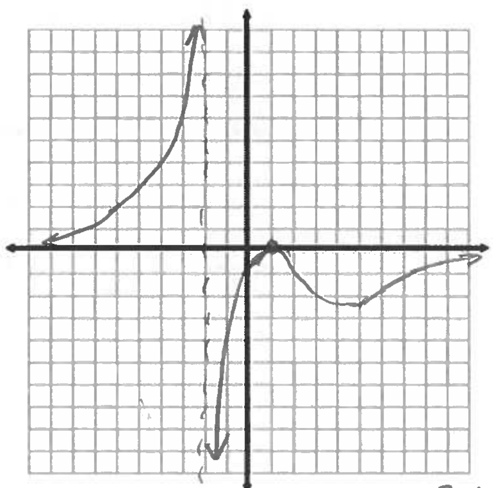
x-int(s)= $x=-1$, $x=2$

y-int= $y=4$

LT= $-2x^5$

$-2(1)(1)(-2)(1)=4$

40) Graph $r(x) = \dfrac{-3(x-1)(x-1)}{4(x+2)(x^2+1)}$

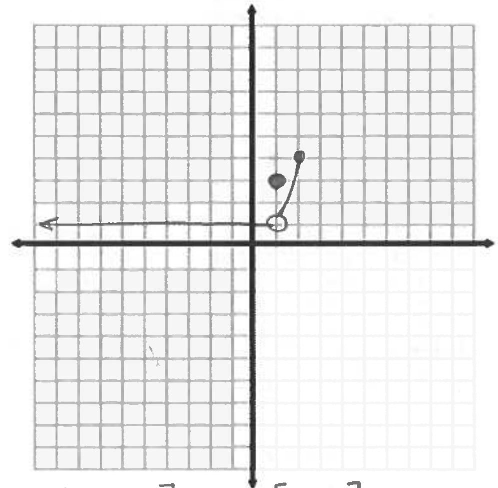


x-int(s)= $x=1$ y-int= $y=-3/8$
VA= $x=-2$ LT/EB= $-3/4x$

$$\frac{-3(-1)(-1)}{4(2)(1)} = \frac{-3}{8}$$

41) Graph

$$f(x) = \begin{cases} 1 & \text{if } x \in (-\infty, 1) \\ 3 & \text{if } x = 1 \\ x^2 & \text{if } x \in (1, 2] \end{cases}$$



D= $(-\infty, 2]$ R= $[\,,4]$

42) State the x and y intercept for:

$$f(x) = \begin{cases} e^x & \text{if } x \leq 2 \\ x^2 - 9 & \text{if } x > 2 \end{cases}$$

$f(0) = e^0 = 1 \Rightarrow \boxed{\text{y-int} = 1}$

$f(x) = 0 \Rightarrow e^x = 0$, never

$x^2 - 9 = 0 \Rightarrow x = \pm 3$ only $x = 3 > 2$.

$\Rightarrow$ x-int $\boxed{x=3}$

Algebra

43) Write $\left(\frac{16}{81}\right)^{\frac{3}{4}}$ as a rational number.

$$= \frac{(16)^{3/4}}{(81)^{3/4}} = \frac{(16^{1/4})^3}{(81^{1/4})^3} = \frac{((2^4)^{1/4})^3}{((3^4)^{1/4})^3} = \frac{2^3}{3^3} = \boxed{\frac{8}{27}}$$

44) Write $7^{-4}\sqrt[2]{7^9}\left(7^{\frac{1}{2}}\right)^3$ as an integer.

$$= 7^{-4} \cdot 7^{\frac{9}{2}} \cdot 7^{\frac{3}{2}} = 7^{-4+\frac{9}{2}+\frac{3}{2}} = 7^{-4+\frac{12}{2}} = 7^{-4+6} = 7^2$$

$$= \boxed{49}$$

45) Write $\log_3\left(\sqrt[5]{\frac{1}{81}}\right)$ as a rational number.

$$\log_3\left(\sqrt[5]{\frac{1}{81}}\right) = \log_3\left(\left(\frac{1}{81}\right)^{1/5}\right) = \frac{1}{5}\log_3\left(\frac{1}{81}\right)$$

$$= \frac{1}{5}\log_3\left(3^{-4}\right) = \frac{-4}{5}\log_3(3) = \boxed{\frac{-4}{5}}$$

46) Write 7^0 as an integer.

$$7^0 = \boxed{1}$$

47) Write $\log_7(1)$ as an integer.

$$\log_7(1) = \boxed{0}$$

48) Find x where $x^3\left(\frac{1}{2}x+3\right)^3 = 8$

$$x^3\left(\frac{1}{2}x+3\right)^3 = \left(x\left(\frac{1}{2}x+3\right)\right)^3 = 8$$

$$\Rightarrow x\left(\frac{1}{2}x+3\right) = 8^{1/3} = 2$$

$$\frac{1}{2}x^2 + 3x - 2 = 0 \Rightarrow$$

$$x = \frac{-3 \pm \sqrt{9-4(\frac{1}{2})(-2)}}{2(\frac{1}{2})}$$

$$= \frac{-3 \pm \sqrt{13}}{1}$$

$$\boxed{x = -3 \pm \sqrt{13}}$$

49) Find x where $2\left(\frac{e^{2x}}{e^{x+3}}\right) + 5 = 7$

$$2\left(\frac{e^{2x}}{e^{x+3}}\right) = 2$$

$$\frac{e^{2x}}{e^{x+3}} = 1 \rightarrow e^{2x-(x+3)} = 1$$

$$e^{x-3} = 1 = e^0 \Rightarrow x - 3 = 0$$

$$\boxed{x = 3}$$

50) Find x where $(2^{3x-5})(3) = \frac{5}{2^{1-x}}$

$$2^{1-x} \cdot 2^{3x-3} = \frac{5}{3}$$

$$2^{1-x+3x-3} = \frac{5}{3}$$

$$2^{2x-2} = \frac{5}{3}$$

$$\log_2\left(2^{2x-2}\right) = \log_2\left(\frac{5}{3}\right)$$

$$2x-2 = \log_2\left(\frac{5}{3}\right)$$

$$\boxed{x = \frac{1}{2}\log_2\left(\frac{5}{3}\right) + 1}$$

51) Find x where $\log_2(x) - \log_2(1-x) = 3$

$$\Rightarrow \log_2\left(\frac{x}{1-x}\right) = 3$$

$$\Rightarrow \cancel{2}^{\log_2\left(\frac{x}{1-x}\right)} = 2^3$$

$$\frac{x}{1-x} = 8$$

$$\Rightarrow x = 8(1-x)$$

$$x = 8 - 8x$$

$$9x = 8$$

$$\boxed{x = \frac{8}{9}}$$

52) Find x where $4\log_e(x) + \log_e(x^3) + 8 = 11$

$$4\log_e(x) + \log_e(x^3) = 3$$

$$\log_e(x^4) + \log_e(x^3) = 3$$

$$\log_e(x^4 \cdot x^3) = 3$$

$$\cancel{e}^{\log_e(x^7)} = e^3$$

$$x^7 = e^3 \Rightarrow x = \sqrt[7]{e^3}$$

$$\boxed{x = e^{3/7}}$$

53) Find x where $2 = 3\log_8(x-1)$

$$\cancel{8}^{\log_8(x-1)} = 8^{\frac{2}{3}}$$

$$x - 1 = 8^{2/3} = (2^3)^{\frac{2}{3}} = 2^2 = 4$$

$$x - 1 = 4 \rightarrow \boxed{x = 5}$$

54) Find $g \circ f(x)$ if $f(x) = x + 3$ and $g(x) = x^2$

$$g(f(x)) = g(x+3) = (x+3)^2 = \boxed{x^2 + 6x + 9}$$

55) Find the inverse of $g(x) = 2\sqrt[3]{4-x}$

$$y = 2\sqrt[3]{4-x}$$

switch x, y

$$\Rightarrow x = 2\sqrt[3]{4-y}$$

$$\sqrt[3]{4-y} = \frac{x}{2}$$

$$\Rightarrow 4 - y = \left(\frac{x}{2}\right)^3 = \frac{x^3}{8}$$

$$y = \boxed{g^{-1}(x) = 4 - \frac{x^3}{8}}$$

56) Find the inverse of $m(x) = 3\log_e(5-2x)$

$y = 3\log_e(5-2x)$

switch x, y $\Rightarrow$ $x = 3\log_e(5-2y)$

$\log_e(5-2y) = \frac{x}{3}$

$\Rightarrow \cancel{e}^{\log_e(5-2y)} = e^{x/3}$

$5-2y = e^{x/3}$

$2y = 5 - e^{x/3}$

$\boxed{y = m^{-1}(x) = \frac{5}{2} - \frac{1}{2}e^{x/3}}$

57) Find the implied domain for $p(x) = \sqrt[2]{4-x}$

no even roots of negatives

$\Rightarrow$ we need $4 - x \geq 0$ $\Rightarrow \boxed{\text{domain} = (-\infty, 4]}$

$\Rightarrow \boxed{x \leq 4}$

58) Find the implied domain for $a(x) = \dfrac{x+1}{x^2-4}$

can't divide by 0

$\Rightarrow \boxed{\text{Domain} = \mathbb{R} - \{2, -2\}}$

$\Rightarrow x^2 - 4 = 0$

$x = \pm 2$

59) Find the implied domain of $f(x) = x^2 - 3x + \log_e(4-5x)$

no negatives or 0 input into a logarithm

$\Rightarrow 4 - 5x > 0$ $\quad x < 4/5$ $\Rightarrow \boxed{\text{Domain} = (-\infty, 4/5)}$

$5x < 4$

60) Find the implied domain of $g(x) = \frac{x^3}{2} - 7\sqrt[3]{x-4}$

No problems $\Rightarrow \boxed{\mathbb{R}}$

61) Complete the square: write $2x^2 - 4x - 3$ in the form of $\alpha(x+\beta)^2 + \gamma$ where $\alpha, \beta, \gamma \in \mathbb{R}$.

$a = 2$

$b = -4$

$c = -3$

$$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$= 2(x-1)^2 + -3 - \frac{16}{8}$$

$$= \boxed{2(x-1)^2 - 5}$$

62) How many roots does $2x^2 - 3x + 4$ have?

Discriminant $\quad b^2 - 4ac = (-3)^2 - 4(2)(4)$

$a = 2,\ b = -3$ $\quad = 9 - 32 = -23 < 0$

$c = 4$ $\quad \Rightarrow \boxed{\text{no real roots}}$

63) Find the roots of $x^2 - 5x + 2$

$a = 1$

$b = -5$

$c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4(1)(2)}}{2(1)} = \boxed{\frac{5 \pm \sqrt{17}}{2}}$$

64) Find a root of $x^3 + 2x^2 - x + 6$

$f(x) = x^3 + 2x^2 - x + 6$

$f(-1) = (-1)^3 + 2(-1)^2 + 1 + 6$

$= -1 + 2 + 1 + 6 = 8$

$f(-2) = (-2)^3 + 2(-2)^2 - (-2) + 6 = -8 + 8 + 2 + 6 = 8$

$f(-3) = (-3)^3 + 2(-3)^2 - (-3) + 6$

$= -27 + 18 + 3 + 6 = 0$ ✓

$\Rightarrow \boxed{x = -3}$ is a root

65) Completely factor: $f(x) = x^3 + 2x^2 - x + 6$ (Hint: -3 is a root)

$x = -3$ is a root, then $(x + 3)$ is a factor

$$\begin{array}{r|l} & x^2 - x + 2 \\ x + 3 & x^3 + 2x^2 - x + 6 \\ & -(x^3 + 3x^2) \\ & \quad -x^2 - x + 6 \\ & \quad -(-x^2 - 3x) \\ & \qquad 2x + 6 \end{array}$$

$\Rightarrow x^3 + 2x^2 - x + 6 = (x + 3)(x^2 - x + 2)$

Discriminant $\quad b^2 - 4ac = (-1)^2 - 4(1)(2)$

$= -7 < 0$

$\Rightarrow$ no more real roots

$\Rightarrow \boxed{(x + 3)(x^2 - x + 2)}$

66) Solve for x if $|2x - 3| + 2 < 10$

$|2x - 3| < 8$

$2x - 3 < 8$

$2x < 11$

$x < 11/2$

$-(2x - 3) < 8$

$2x - 3 > -8$

$2x > -5$

$x > -5/2$

$$\boxed{-\frac{5}{2} < x < \frac{11}{2}}$$

67) Solve for x if $|3x-2|-5 \geq 1$

$$|3x-2| \geq 6$$

$$3x-2 \geq 6 \qquad -(3x-2) \geq 6$$

$$3x \geq 8 \qquad 3x-2 \leq -6$$

$$\boxed{x \geq 8/3} \qquad 3x \leq -4$$

$$\boxed{x \leq -4/3}$$

Linear Algebra

68) Find the determinant of the matrix: $\begin{pmatrix} -2 & -3 \\ 1 & 0 \end{pmatrix}$

$$= -2 \cdot 0 - (-3)(1) = 0 - -3 = \boxed{3}$$

69) Find the determinant of the matrix: $\begin{pmatrix} -2 & -1 & 1 \\ 0 & -1 & 2 \\ -1 & 0 & 3 \end{pmatrix}$

$$-2 \det \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} - (-1) \det \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} + 1 \det \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= -2(-1 \cdot 3 - 2 \cdot 0) + (0 \cdot 3 - (-1)2) + (0 \cdot 0 - (-1)(-1))$$

$$= -2(-3) + 2 - 1 = \boxed{7}$$

70) Find the product of $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$$= \begin{bmatrix} 1 \cdot 1 + 0 \cdot 0 & 1 \cdot 2 + 0 \cdot 1 \\ 3 \cdot 1 + 1 \cdot 0 & 3 \cdot 2 + 1 \cdot 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}}$$

71) Find the inverse of $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$$\det \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = 1 \cdot 3 - 4 \cdot 2 = 3 - 8 = -5$$

$$A^{-1} = \frac{1}{-5} \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -3/5 & 4/5 \\ 2/5 & -1/5 \end{bmatrix}}$$

72) Find $\begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}^{-1}$

$$\det \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix} = -1\cdot 4 - 2\cdot -3 = -4-(-6) = -4+6 = 2$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ 3 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & -1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$

73) Find $\det \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix}$

$$= 1\cdot \det\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} - (-2)\det\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} + -3\det\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= 1(2) + 2(1) - 3(-1) = 2+2+3 = \boxed{7}$$

74) Write the following system of three linear equations in three variables as a matrix equation:

$2x - y + z = 2$
$y + 2x = 1$
$-x + y - z = 0$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 1 \\ 2 & 1 & 0 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

75) Solve for x, y, z if:

$$\underset{A}{\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}}\underset{\vec{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{b}{\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}} \text{ and } \underset{A^{-1}}{\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1}} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix}$$

not actually the inverse matrix... oops

$$\Rightarrow \quad A\vec{x} = b$$

$$A^{-1}A\vec{x} = A^{-1}b$$

$$\vec{x} = A^{-1}b \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1\cdot -2 + -1\cdot 2 + 0\cdot 1 \\ -1\cdot -2 + 2\cdot 2 + 1\cdot 1 \\ -4\cdot -2 + 5\cdot 2 + 2\cdot 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ 20 \end{bmatrix} \quad \boxed{\begin{matrix} x=-4 \\ y=7 \\ z=20 \end{matrix}}$$