

Rules for Numbers

The real numbers are governed by a collection of rules that have to do with addition, multiplication, and inequalities. In the rules below, $x, y, z \in \mathbb{R}$. (In other words, x, y , and z are real numbers.)

Rules of addition.

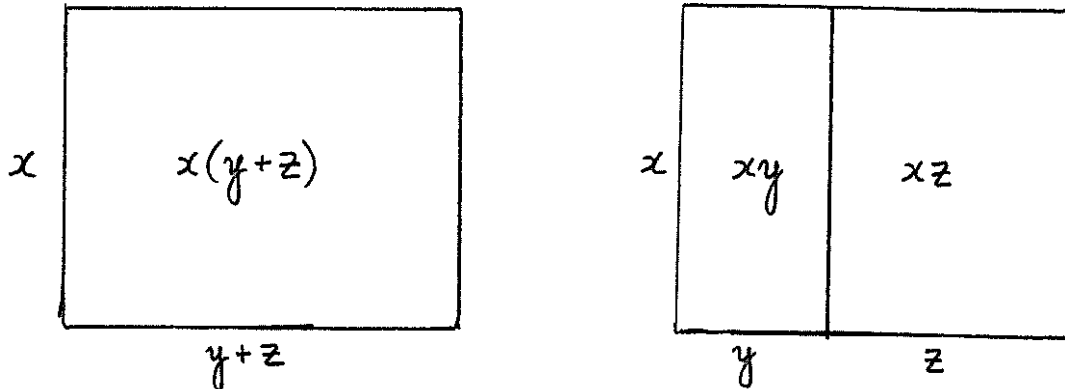
- $(x + y) + z = x + (y + z)$ (Law of associativity)
- $x + y = y + x$ (Law of commutativity)
- $x + 0 = x$ (Law of identity)
- $-x + x = 0$ (Law of inverses)

Rules of multiplication.

- $(xy)z = x(yz)$ (Law of associativity)
- $xy = yx$ (Law of commutativity)
- $x1 = x$ (Law of identity)
- If $x \neq 0$ then $\frac{1}{x}x = 1$ (Law of inverses)

Distributive Law. There is a rule that combines addition and multiplication: the distributive law. Of all the rules listed so far, it's arguably the most important.

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- $x(y + z) = xy + xz$ (Distributive Law)
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Here are some other forms of the distributive law that you will have to be comfortable with:

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- $(y + z)x = yx + zx$
 - $x(y - z) = xy - xz$
 - $(x + y)(z + w) = xz + xw + yz + yw$
 - $x(y + z + w) = xy + xz + xw$
 - $x(y_1 + y_2 + y_3 + \cdots + y_n) = xy_1 + xy_2 + xy_3 + \cdots + xy_n$
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Examples. Sometimes you'll have to use the distributive law in the "forwards" direction, as in the following three examples:

- $3(y + z) = 3y + 3z$
- $(-2)(4y - 5z) = (-2)4y - (-2)5z = -8y + 10z$
- $2(3x - 2y + 4z) = 6x - 4y + 8z$

Sometimes you'll have to use the distributive law in "reverse". This process is sometimes called *factoring out a term*. The three equations below are examples of factoring out a -4 , factoring out a 3 , and factoring out a 2 .

- $-4y - 4z = -4(y + z)$
- $3x + 6y = 3(x + 2y)$
- $10x - 8y + 4z = 2(5x - 4y + 2z)$

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In addition to the algebra rules above, the real numbers are governed by laws of inequalities.

Rules of inequalities.

- If $x > 0$ and $y > 0$ then $x + y > 0$
- If $x > 0$ and $y > 0$ then $xy > 0$
- If $x \in \mathbb{R}$, then either $x > 0$, or $x < 0$, or $x = 0$

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Intervals

The following chart lists 8 important types of subsets of \mathbb{R} . Any set of one of these types is called an *interval*.

Name of set	Those $x \in \mathbb{R}$ contained in the set
$[a, b]$	$a \leq x \leq b$
(a, b)	$a < x < b$
$[a, b)$	$a \leq x < b$
$(a, b]$	$a < x \leq b$
$[a, \infty)$	$a \leq x$
$(-\infty, b]$	$x \leq b$
(a, ∞)	$a < x$
$(-\infty, b)$	$x < b$

Notice that for every interval listed in the chart above, the least of the two numbers written in the interval is always written on the left, just as they appear in the real number line. For example, $3 < 7$, so 3 is drawn on the left of 7 in the real number line, and the following intervals are legitimate intervals to write: $[3, 7]$, $(3, 7)$, $[3, 7)$, and $(3, 7]$. You must *not* write an interval such as $(7, 3)$, because the least of the two numbers that define an interval has to be written on the left.

Similarly, $(\infty, 2)$ or $[5, -\infty)$ are *not* proper ways of writing intervals.

Exercises

Decide whether the following statements are true or false.

1) $5x + 5y = 5(x + y)$

2) $3x + y = 3(x + y)$

3) $4x - 6y = 2(2x - 3y)$

4) $x + 7y = 7(x + y)$

5) $36x - 9y + 81z = 3(12x - 3y + 9z)$

6) $2 \in (2, 5]$

7) $0 \in (-4, 0]$

8) $-3 \in [-3, 1)$

9) $156, 345, 678 \in (-1, \infty)$

10) $2 \in (-\infty, -3]$

11) $[7, 10) \subseteq [7, 10]$

12) $[-17, \infty) \subseteq (-17, \infty)$

13) $(-4, 0] \subseteq [-4, 0)$

14) $(-\infty, 20] \subseteq (-\infty, -7]$

15) $[0, \infty) \subseteq \mathbb{R} - \{\pi\}$

16) $\{3, 10, 7\} \subseteq (2, 8)$

17) $\{0, 2, \frac{4}{5}, \sqrt{2}\} \subseteq [0, \infty)$