Constant & Linear Polynomials

Constant polynomials

A *constant polynomial* is the same thing as a constant function. That is, a constant polynomial is a function of the form

$$p(x) = c$$

for some number c. For example, $p(x) = -\frac{5}{3}$ or $q(x) = -7$.

The output of a constant polynomial does not depend on the input (notice that there is no x on the right side of the equation $p(x) = c$). Constant polynomials are also called degree 0 polynomials.

The graph of a constant polynomial is a horizontal line. A constant polynomial does not have any roots unless it is the polynomial $p(x) = 0$.

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Linear polynomials

A linear polynomial is any polynomial defined by an equation of the form

$$p(x) = ax + b$$

where a and b are real numbers and $a \neq 0$. For example, $p(x) = 3x - 7$ and $q(x) = \frac{-13}{4}x + \frac{5}{3}$ are linear polynomials. A linear polynomial is the same thing as a degree 1 polynomial.

Roots of linear polynomials

Every linear polynomial has exactly one root. Finding the root is just a matter of basic algebra.

Problem: Find the root of $p(x) = 3x - 7$.

Solution: The root of $p(x)$ is the number α such that $p(\alpha) = 0$. In this problem that means that $3\alpha - 7 = 0$. Hence $3\alpha = 7$, so $\alpha = \frac{7}{3}$. Thus, $\frac{7}{3}$ is the root of $3x - 7$.

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Slope

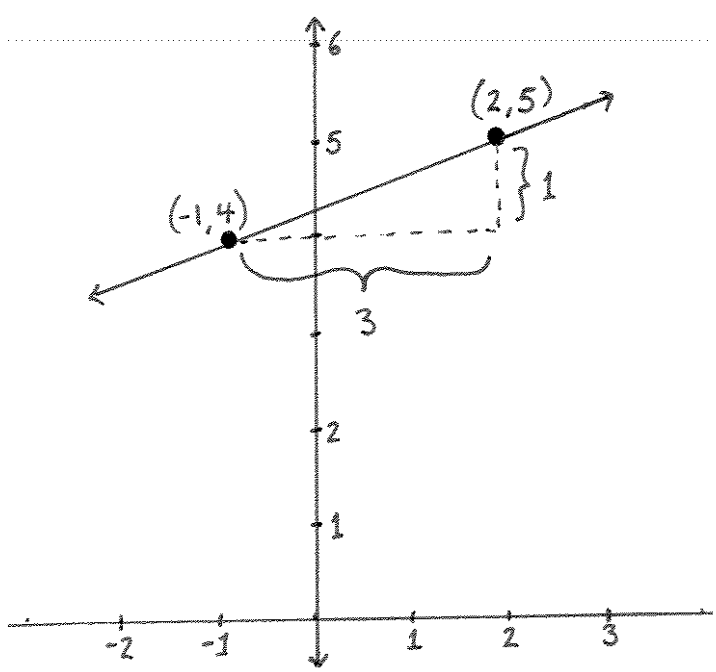
The slope of a line is the ratio of the change in the second coordinate to the change in the first coordinate. In different words, if a line contains the two points (x_1, y_1) and (x_2, y_2), then the slope is the change in the y-coordinate – which equals $y_2 - y_1$ – divided by the change in the x-coordinate – which equals $x_2 - x_1$.

Slope of line containing (x_1, y_1) and (x_2, y_2):

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Example: The slope of the line containing the two points $(-1, 4)$ and $(2, 5)$ equals

$$\frac{5 - 4}{2 - (-1)} = \frac{1}{3}$$



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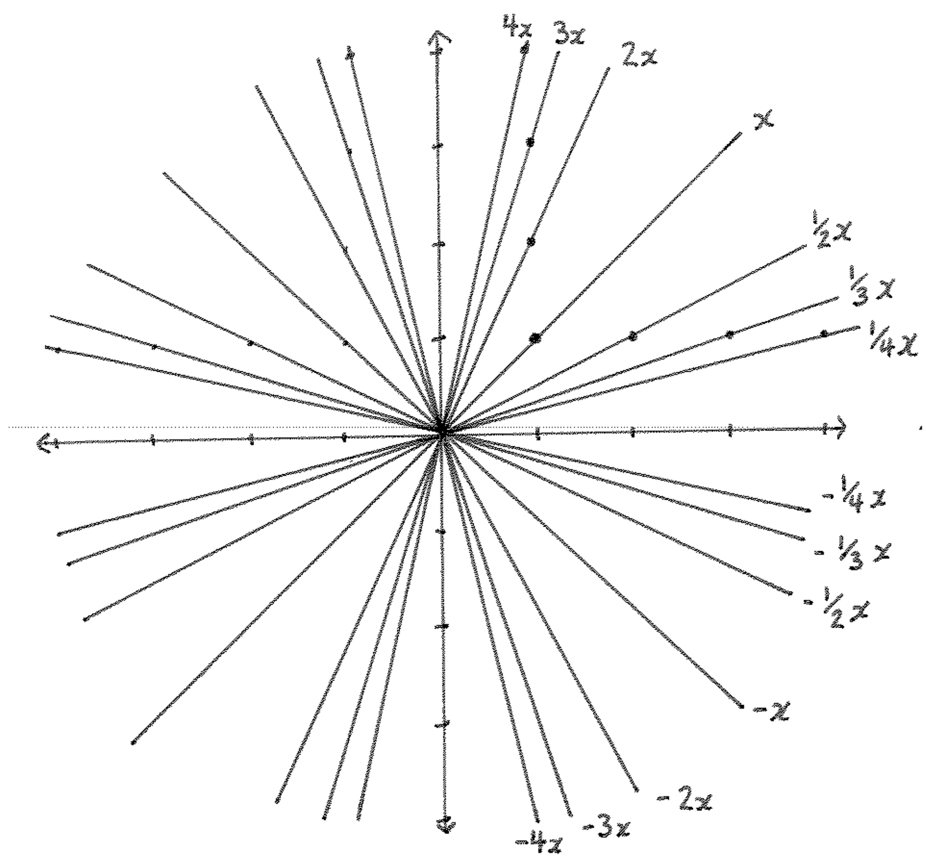
Graphing linear polynomials

Let $p(x) = ax$ where a is a number that does not equal 0. This polynomial is an example of a linear polynomial.

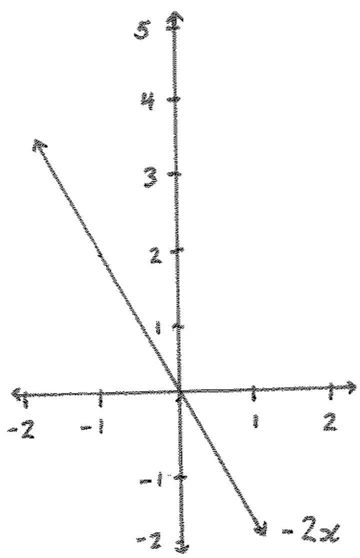
The graph of $p(x) = ax$ is a straight line that passes through $(0, 0) \in \mathbb{R}^2$ and has slope equal to a. We can check this by graphing it. The point $(0, a0) = (0, 0)$ is in the graph, as are the points $(1, a)$, $(2, 2a)$, $(3, 3a)$,... and $(-1, -a)$, $(-2, -2a)$, $(-3, -3a)$,...

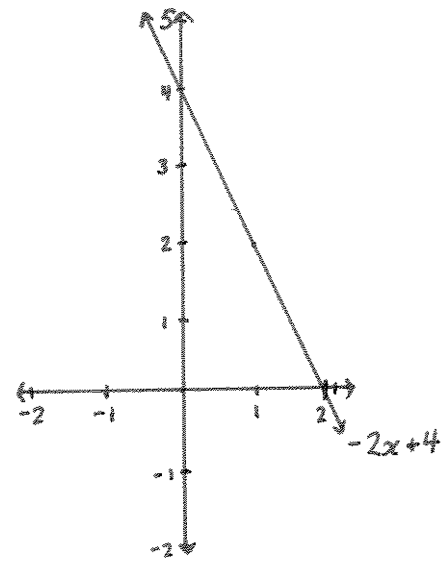


Because the graph of $ax + b$ is the graph of ax shifted up or down by b – depending on whether b is positive or negative – the graph of $ax + b$ is a straight line that passes through $(0, b) \in \mathbb{R}^2$ and has slope equal to a.

Problem: Graph $p(x) = -2x + 4$.

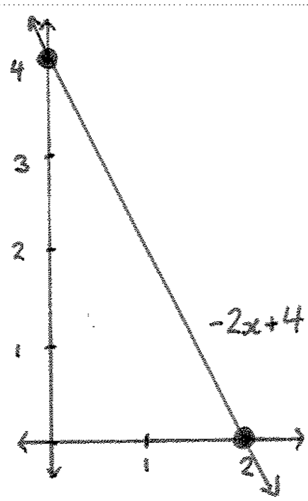
Solution: The graph of $-2x+4$ is the graph of $-2x$ "shifted up" by 4. Draw $-2x$, which is the line of slope -2 that passes through $(0, 0)$, and then shift it up to the line that passes through $(0, 4)$ and is parallel to $-2x$.





Another solution: To graph a linear polynomial, find two points in the graph, and then draw the straight line that passes through them.

Since $p(x) = -2x + 4$ has 2 as a root, it has an x-intercept at 2. The y-intercept is the point in the graph whose first coordinate equals 0, and that's the point $(0, p(0)) = (0, 4)$. To graph $-2x + 4$, draw the line passing through the x- and y-intercepts.



Behind the name. Degree 1 polynomials are called linear polynomials because their graphs are straight lines.

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Exercises

1.) Graph $p(x) = 3$.

2.) Graph $q(x) = -\frac{3}{2}$.

3.) Find the root of $p(x) = -\frac{4}{3}x + \frac{6}{7}$.

4.) Find the root of $q(x) = \frac{2}{9}x - \frac{8}{5}$.

5.) Plot the x- and y-intercepts of $p(x) = 4x - 3$, and then graph $p(x)$.

6.) Plot the x- and y-intercepts of $q(x) = -2x - 3$, and then graph $q(x)$.

7.) Claudia owns a coconut collecting company. She has to pay \$200 for a coconut collecting license to conduct her company, and she earns \$3 for every coconut she collects. If x is the number of coconuts she collects, and $p(x)$ is the number of dollars her company earns, then find an equation for $p(x)$.

8.) Spencer is payed \$400 to collect coconuts no matter how many coconuts he collects. Because he is collecting coconuts for a flat fee, the local government does not require Spencer to purchase a coconut collecting license. If $q(x)$ is the number of dollars he earns for collecting x coconuts, what is the equation that defines $q(x)$?

9.) If Claudia and Spencer collect the same number of coconuts, then how many coconuts would Claudia have to collect for her company to earn at least as much money as Spencer?