Sets & Numbers

Sets

A *set* is a collection of objects. For example, the set of days of the week is a set that contains 7 objects: Mon., Tue., Wed., Thur., Fri., Sat., and Sun..

Set notation. Writing $\{2, 3, 5\}$ is a shorthand for the set that contains the numbers 2, 3, and 5, and no objects other than 2, 3, and 5.

The order in which the objects of a set are written doesn't matter. For example, $\{5, 2, 3\}$ and $\{2, 3, 5\}$ are the same set. Alternatively, the previous sentence could be written as "For example, $\{5, 2, 3\} = \{2, 3, 5\}$."

If B is a set, and x is an object contained in B, we write $x \in B$. If x is not contained in B then we write $x \notin B$.

Examples.

- $5 \in \{2, 3, 5\}$
- $1 \notin \{2, 3, 5\}$

Subsets. One set is a *subset* of another set if every object in the first set is an object of the second set as well. The set of weekdays is a subset of the set of days of the week, since every weekday is a day of the week.

A more succinct way to express the concept of a subset is as follows:

The set B is a *subset* of the set C if every $b \in B$ is also contained in C.

Writing $B \subseteq C$ is a shorthand for writing "B is a subset of C". Writing $B \nsubseteq C$ is a shorthand for writing "B is *not* a subset of C".

Examples.

- $\{2, 3\} \subseteq \{2, 3, 5\}$
- $\{2, 3, 5\} \nsubseteq \{3, 5, 7\}$

Set minus. If A and B are sets, we can create a new set named $A - B$ (spoken as "A minus B") by starting with the set A and removing all of the objects from A that are also contained in the set B.

Examples.

- $\{1,7,8\} - \{7\} = \{1,8\}$
- $\{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\} = \{1,3,5,7,9\}$

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Numbers

Among the most common sets appearing in math are sets of numbers. There are many different kinds of numbers. Below is a list of those that are most important for this course.

Natural numbers. $\mathbb{N} = \{1, 2, 3, 4, ...\}$

Integers. $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, 3, ...\}$

Rational numbers. $\mathbb{Q}$ is the set of fractions of integers. That is, the numbers contained in $\mathbb{Q}$ are exactly those of the form $\frac{n}{m}$ where n and m are integers and $m \neq 0$.

For example, $\frac{1}{3} \in \mathbb{Q}$ and $\frac{-7}{12} \in \mathbb{Q}$.

Real numbers. $\mathbb{R}$ is the set of numbers that can be used to measure a distance, or the negative of a number used to measure a distance. The set of real numbers can be drawn as a line called "the number line".

$\sqrt{2}$ and π are two of very many real numbers that are not rational numbers.

(Aside: the definition of $\mathbb{R}$ above isn't very precise, and thus isn't a very good definition. The set of real numbers has a better definition, but it's outside the scope of this course. For this semester we'll make due with this intuitive notion of what a real number is.)

Numbers as subsets. $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

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Exercises

Decide whether the following statements are true or false.

1) $3 \in \{7, 4, -10, 17, 3, 9, 67\}$

2) $4 \in \{14, 44, 43, 24\}$

3) $\frac{1}{3} \in \mathbb{Z}$

4) $-5 \in \mathbb{N}$

5) $\frac{-271}{113} \in \mathbb{Q}$

6) $-37 \in \mathbb{Z}$

7) $5 \in \mathbb{R} - \{4, 6\}$

8) $\{2, 4, 7\} \subseteq \{-3, 2, 5, 4, 7\}$

9) $\{2, 3, 5\} \subseteq \{2, 5\}$

10) $\{2, 5, 9\} \subseteq \{2, 4, 9\}$

11) $\{-15, \frac{3}{4}, \pi\} \subseteq \mathbb{R}$

12) $\{-15, \frac{3}{4}, \pi\} \subseteq \mathbb{Q}$

13) $\{-2, 3, 0\} \subseteq \mathbb{N}$

14) $\{-2, 3, 0\} \subseteq \mathbb{Z}$

15) $\{\sqrt{2}, 271\} \subseteq \mathbb{R}$

16) $\{\sqrt{2}, 271\} \subseteq \mathbb{Q}$