Solutions

Math 1050-006 Practice Final Exam

1.) Find 3 things that are wrong with the following statement:

$[2,5] \in (2,1)$

① $(2,1)$ should be $(1,2)$

② Wrong contained in symbol, $\in$ should be $\subseteq$

③ $5 \notin (1,2) \Rightarrow [2,5] \not\subseteq (1,2)$

2.) If the function f: $\mathbb{Q} \to \mathbb{R}$ is defined as $f(x) = x^2 + 2x$ then:

a) What set does the object x belong to?

As the domain is $\mathbb{Q}$, $x \in \mathbb{Q}$, ie x is a rational number

b) What is f(3)?

$f(3) = (3)^2 + 2(3) = 9 + 6 = \boxed{15}$

c) What is f(π)?

$\pi \notin \mathbb{Q} \Rightarrow f(\pi)$ is undefined

3.) If $a_1, a_2, a_3, ... = -3, 1, 5, ...$ then what is the 80th term in this sequence, a_{80}?

This looks like an arithmetic sequence, $a_{n+1} = a_n + d$

$\Rightarrow a_2 = 1 = a_1 + d = -3 + d$

$1 = -3 + d$

$d = 4$

The prediction equation of an arithmetic sequence is $a_n = a_1 + (n-1)d$

$n = 80,\ a_1 = -3$ $\Rightarrow$ $a_{80} = -3 + (80-1)4 = -3 + 79(4)$

$d = 4$

$= 316 - 3 = \boxed{313}$

4.) What is $\sum_{k=1}^{20} (k-3)$?

$$\sum_{k=1}^{20} (k-3) = \sum_{k=1}^{20} k + \sum_{k=1}^{20} (-3)$$

$=$ arithmetic sum $+$ constant sum

arithmetic sum $= \frac{n}{2}(a_n + a_1)$

$n = 20$

$a_1 = 1$

$a_{20} = 20$

$= \frac{20}{2}(1 + 20)$

$= 10(21) = 210$

constant sum $= n$(constant)

$n = 20$

constant $= -3$

$= 20(-3)$

$= -60$

$$\Rightarrow \sum_{k=1}^{20} (k-3) = 210 - 60 = \boxed{150}$$

5.) How many different ways can you arrange the 6 letters a,b,c,d,e,f into 'words'?
(for example: bacdef is a 'word')

~~There are 6 objects~~ and order matters

$\Rightarrow$ $\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 6! = \boxed{720}$

6.) Write out Pascal's triangle to the row n=4 and use the Binomial theorem and Pascal's Triangle to factor out $(x+y)^4$

Binomial Theorem: $\sum_{i=0}^{4} \binom{4}{i} x^i y^{(4-i)}$

n=0 1
n=1 1 1
n=2 1 2 1
n=3 1 3 3 1
n=4 1 4 6 4 1

Pascal's Triangle

$$= \binom{4}{0}x^0y^4 + \binom{4}{1}x^1y^3 + \binom{4}{2}x^2y^2 + \binom{4}{3}x^3y^1 + \binom{4}{4}x^4y^0$$

$$\boxed{= y^4 + 4xy^3 + 6x^2y^2 + 4x^3y + x^4}$$

7.) If h(x) = x^2+x and g(x) = x-2 solve for:

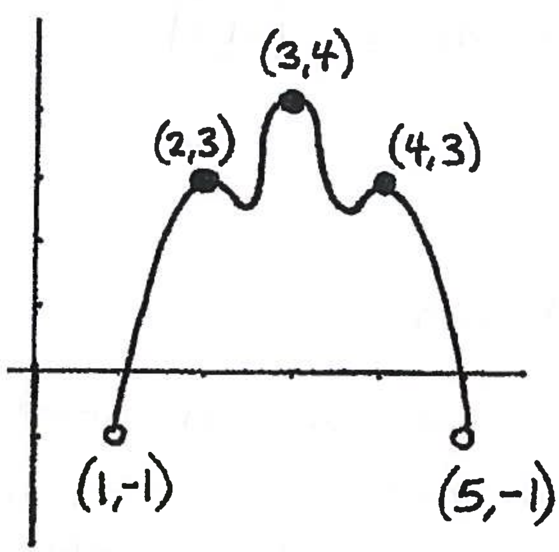
a) $g \circ h(x)$

$$g(h(x)) = g(x^2+x) = (x^2+x) - 2 = \boxed{x^2+x-2}$$

b) $h \circ g(x)$

$$h(g(x)) = h(x-2) = (x-2)^2 + (x-2) = x^2-4x+4+x-2$$
$$= \boxed{x^2-3x+2}$$

8.) Find the domain and range of the following graph:



Domain goes from 1 to 5 with 1,5 not included

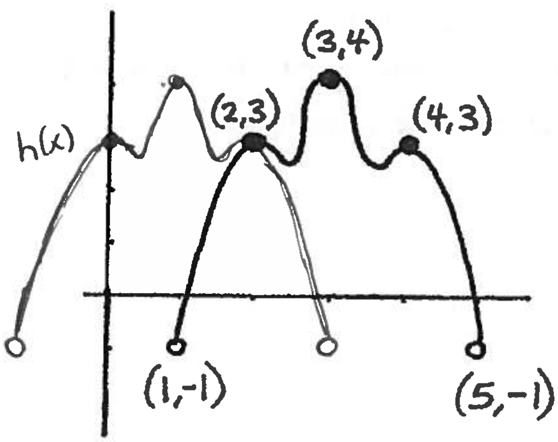
$\Rightarrow$ $\boxed{\text{Domain} = (1,5)}$

Range goes from -1 to 4 with -1 not included, 4 included

$\Rightarrow$ $\boxed{\text{Range} = (-1,4]}$

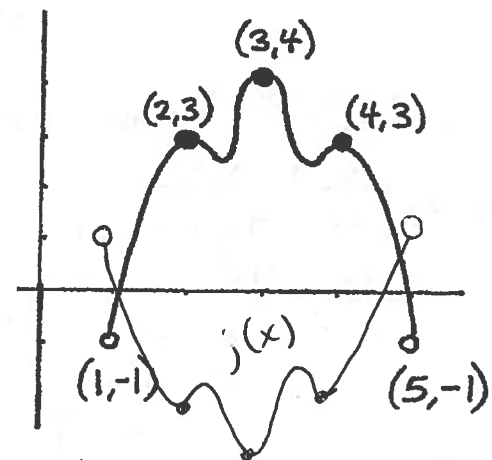
9.) Given the graph from problem 8 of g(x), graph h(x) = g(x+2) and j(x) = - g(x) using graph transformations

a) h(x)



h(x) = before transformation of g(x) to the left by 2

b) j(x)

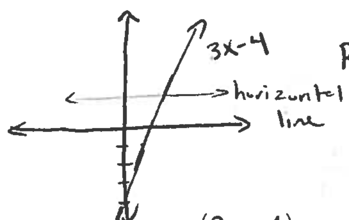


j(x) = after transformation of g(x) flip over x-axis

10.) if f: ℝ→ℝ and f(x) = 3x-4 using the ideas of one-to-one and onto decide whether or not f(x) has an inverse function

one-to-one : passes the horizontal line test ✓

onto : Target = Range ✓



Range = ℝ

Target = ℝ

⇒ Range = Target

one-to-one and onto

⇒ f(x) has an inverse

11.) If $g(x) = \dfrac{(2x-4)}{(x+3)}$, find $g^{-1}(x)$. (*Assuming the implied domain* $x \neq -3$)

$$y = \frac{2x-4}{x+3}$$

$$(x+3)y = 2x-4$$

$$xy + 3y = 2x-4$$

$$xy - 2x = -3y-4$$

$$x(y-2) = -3y-4$$

$$x = \frac{-3y-4}{y-2} = g^{-1}(y)$$

$$\Rightarrow \boxed{g^{-1}(x) = \frac{-3x-4}{x-2} = \frac{3x+4}{2-x}}$$

12.) What is the implied domain of the function $f(x) = 3\sqrt[6]{-2x+4}$?

① can't take even roots of negative numbers

② can't divide by 0

$\boxed{\text{Domain} = \mathbb{R} - (2,\infty)}$

$\Rightarrow -2x+4<0 \qquad x>+2$

$-2x<-4 \qquad \Rightarrow$ we cant have $x>2$

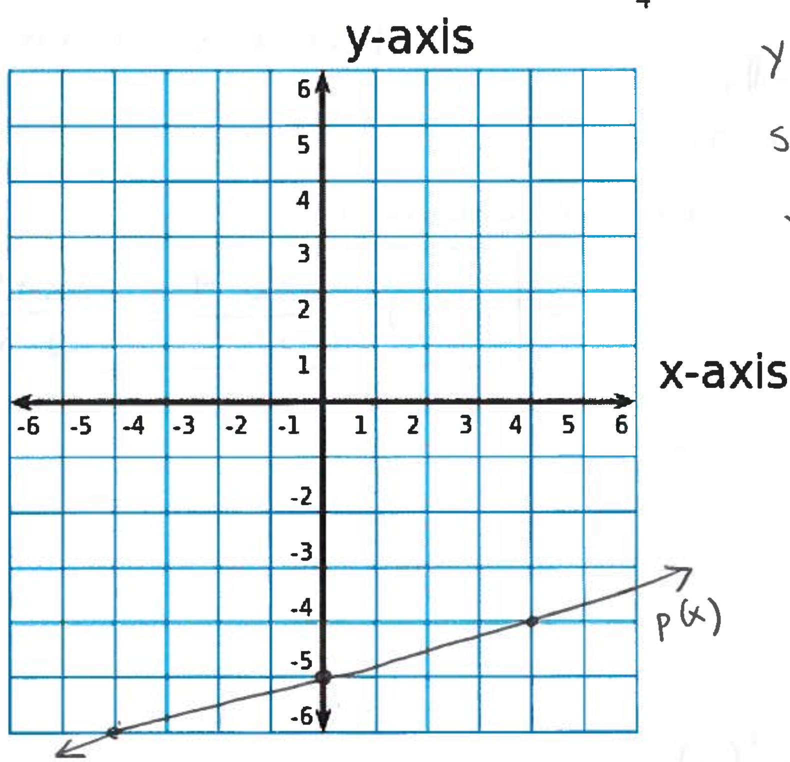
13.) Solve $\dfrac{(2x^3-4x^2+5x-7)}{2x-4}$ express your answer with the remainder if you find one.

$$\begin{array}{r} x^2+0x+\frac{5}{2} \\ 2x-4\,\overline{)\,2x^3-4x^2+5x-7} \\ -(2x^3-4x^2) \\ \hline 0x^2+5x \\ -\,(0x^2+0x) \\ \hline 5x-7 \\ -\,5x-10 \\ \hline 3 \end{array}$$

3 = remainder

$\Rightarrow$ solution: $\boxed{x^2+\frac{5}{2}+\frac{3}{2x-4}}$

14.) Graph the linear polynomial function $p(x) = \frac{1}{4}x - 5$.



$y = \frac{1}{4}x - 5$

slope $= \frac{1}{4}$

y-int $= -5$

x-int: $y = 0 = \frac{1}{4}x - 5$

$\frac{1}{4}x = 5$

$x = 20$

x	y
0	-5
1	-4 3/4
4	-4
-4	-6

15.) By completing the square (formula: $p(x)=a(x+\frac{b}{2a})^2+c-\frac{b^2}{4a}$)

Graph the quadratic polynomial $p(x)=2x^2+4x-5$

$a=2,\ b=4,\ c=-5$

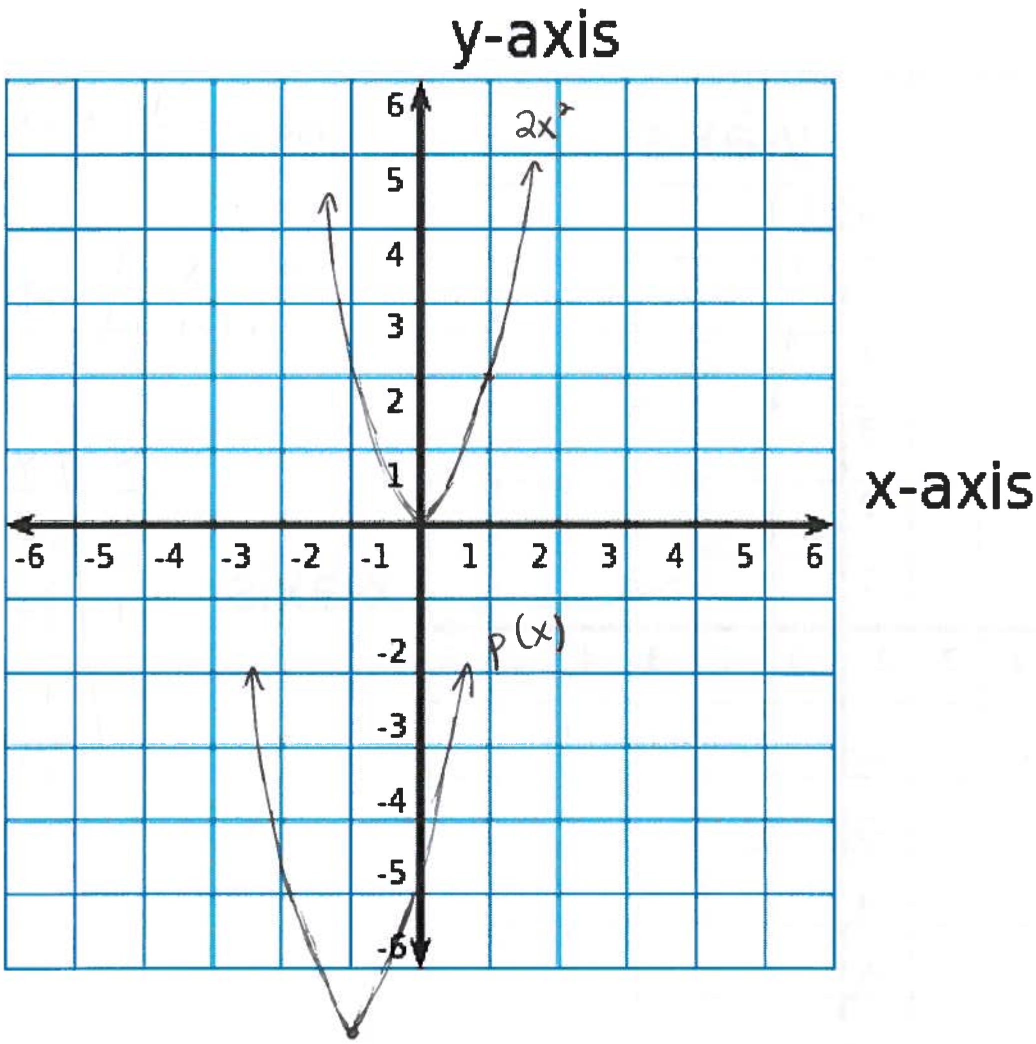
$$p(x) = 2\left(x+\frac{4}{2(2)}\right)^2 + (-5) - \frac{(4)^2}{4(2)}$$

$$= 2(x+1)^2 + -5 - 2$$

$$= 2(x+1)^2 - 7$$

down 7

$\Rightarrow$ graph $2x^2$ left 1



16.) Completely factor the polynomial $p(x)=2x^3+6x^2+2x+6$

$p(x) = 2(x^3+3x^2+x+3)$

$$
\begin{array}{r}
x^2 + 0x + 1 \\
x+3 \,\overline{\big)\, x^3 + 3x^2 + x + 3} \\
-\; x^3 + 3x^2 \\
\hline
0x^2 + x \\
-\; 0x^2 + 0x \\
\hline
x + 3
\end{array}
$$

$\boxed{2(x+3)(x^2+1)}$

guess -1: $2(-1+3-1+3) \neq 0$

$\checkmark -3$: $2(-27+27-3+3)=0$

-3 is a root

$\Rightarrow x+3$ is a factor

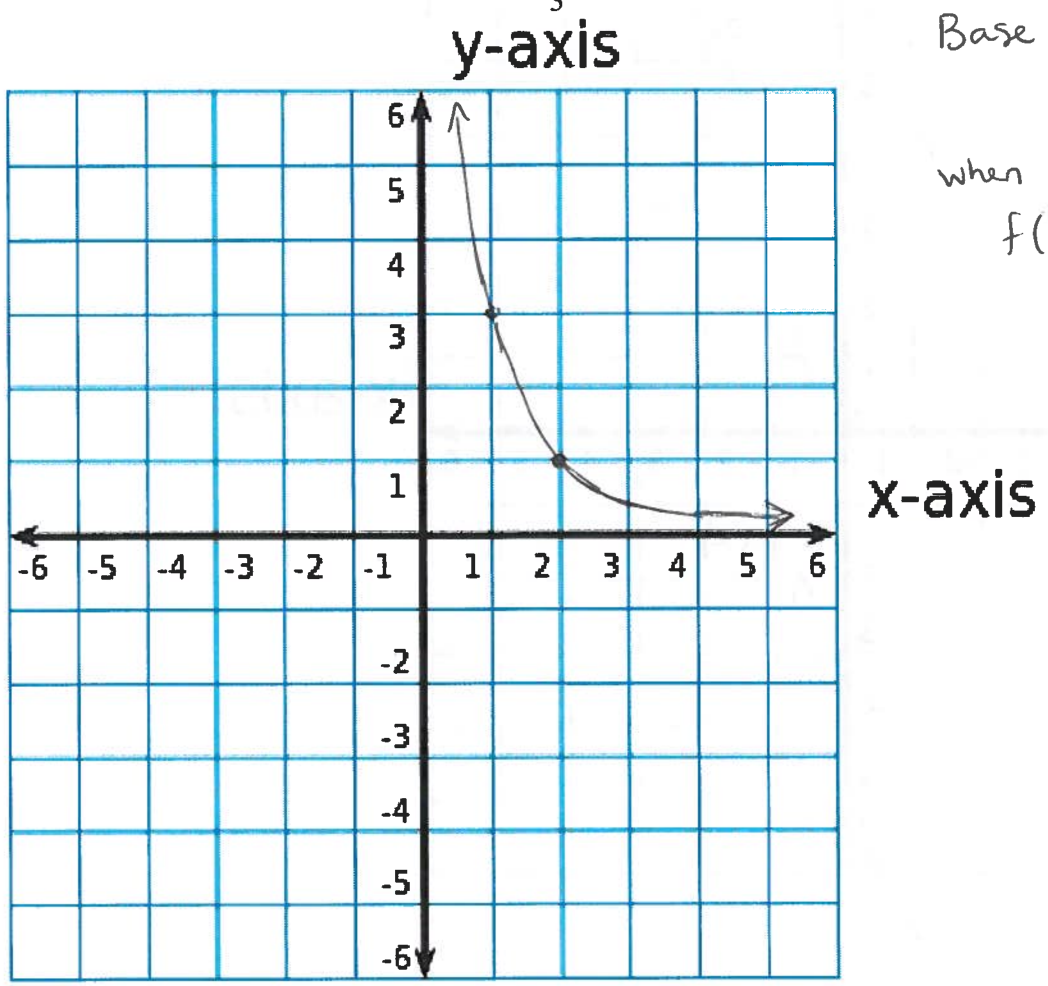
Discriminant x^2+1 = $b^2-4ac = 0^2-4(1)(1)$

$= -4 < 0$

$\Rightarrow$ no roots

x^2+1 is monic

17.) Graph the exponential function $f(x)=(\frac{1}{3})^{(x-2)}$



Base $= \frac{1}{3} \Rightarrow$ exponential decay

when $x=2$

$f(x) = \left(\frac{1}{3}\right)^{2-2} = \left(\frac{1}{3}\right)^0 = 1$

x	y
0	$\left(\frac{1}{3}\right)^{-2} = 9$
1	$\left(\frac{1}{3}\right)^{-1} = 3$
2	1
3	$\left(\frac{1}{3}\right)^{1} = \frac{1}{3}$

18.) Find the vertical assymptotes, x-intercepts, and the leading order term of the rational function

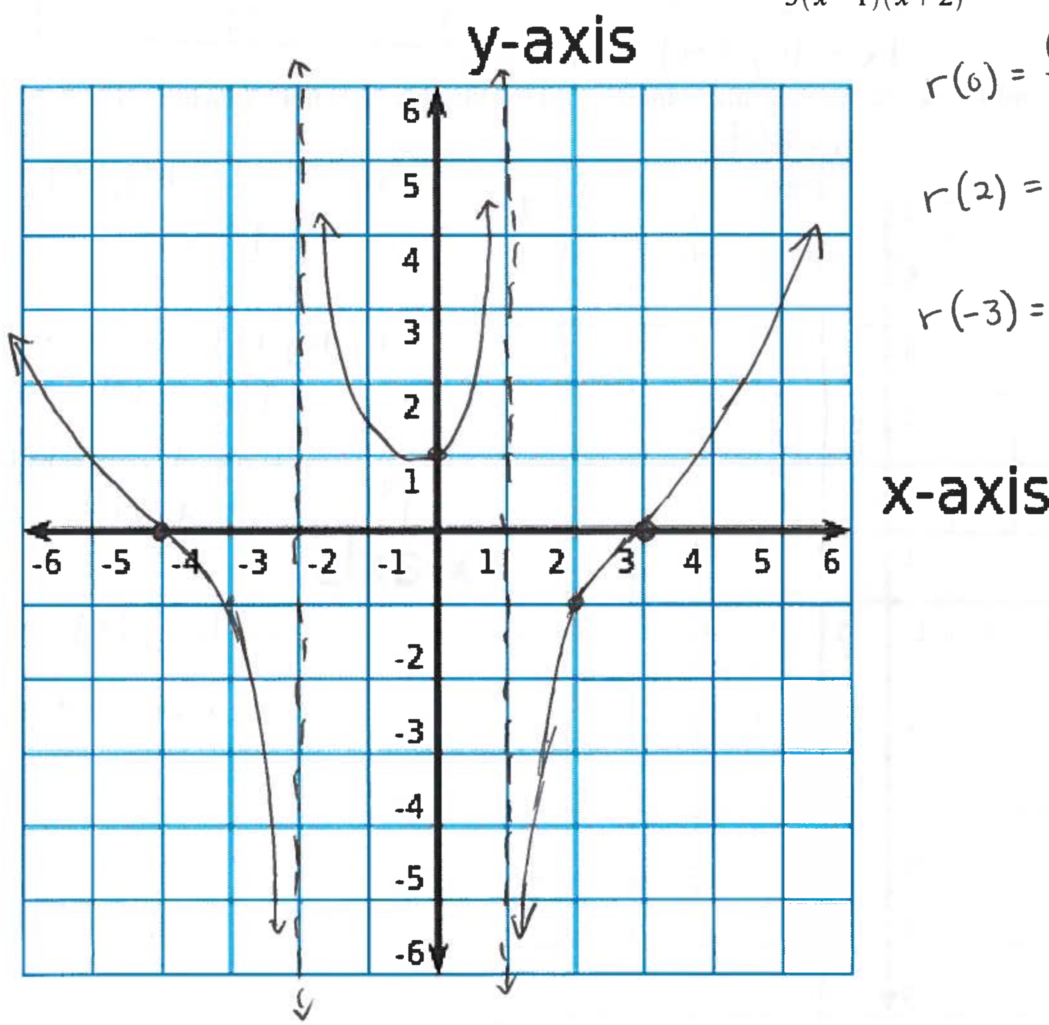
$$r(x)=\frac{(x+4)(x-3)(x^2+1)}{3(x-1)(x+2)}$$

vertical assymptotes: roots of denominator = $\boxed{1,-2}$

x-intercepts = roots of numerator = $\boxed{-4, 3}$

Leading order term = $\dfrac{\text{LOT (numerator)}}{\text{LOT (denominator)}} = \dfrac{(x)(x)(x^2)}{3(x)(x)} = \dfrac{x^4}{3x^2} = \boxed{\dfrac{x^2}{3}}$

19.) Using your solutions from problem 18, graph $r(x)=\frac{(x+4)(x-3)(x^2+1)}{3(x-1)(x+2)}$



$r(0) = \dfrac{(+)(-)(+)}{(+)(-)(+)} = \dfrac{(-)}{(-)} = +$

$r(2) = \dfrac{(+)(-)(+)}{(+)(+)(+)} = \dfrac{(-)}{(+)} = (-$

$r(-3) = \dfrac{(+)(-)(+)}{(+)(-)(-)} = \dfrac{(-)}{(+)} = -$

20.) Solve the logarithmic equation: $\log_3((x+2)^3)+\log_3(9)=5$ for x

$3\log_3(x+2) + \log_3(3^2) = 5$

$3\log_3(x+2) + 2\log_3(3) = 5$

$3\log_3(x+2) = 5-2 = 3$

$\log_3(x+2) = 1$

$3 \quad 3$

$x+2 = 3^1 = 3$

$\boxed{x = 1}$

21.) Solve the exponential equation: $e^x e^{(3x-4)}=4$ for x

$e^x e^{3x-4} = e^{x+3x-4} = e^{4x-4} = 4$

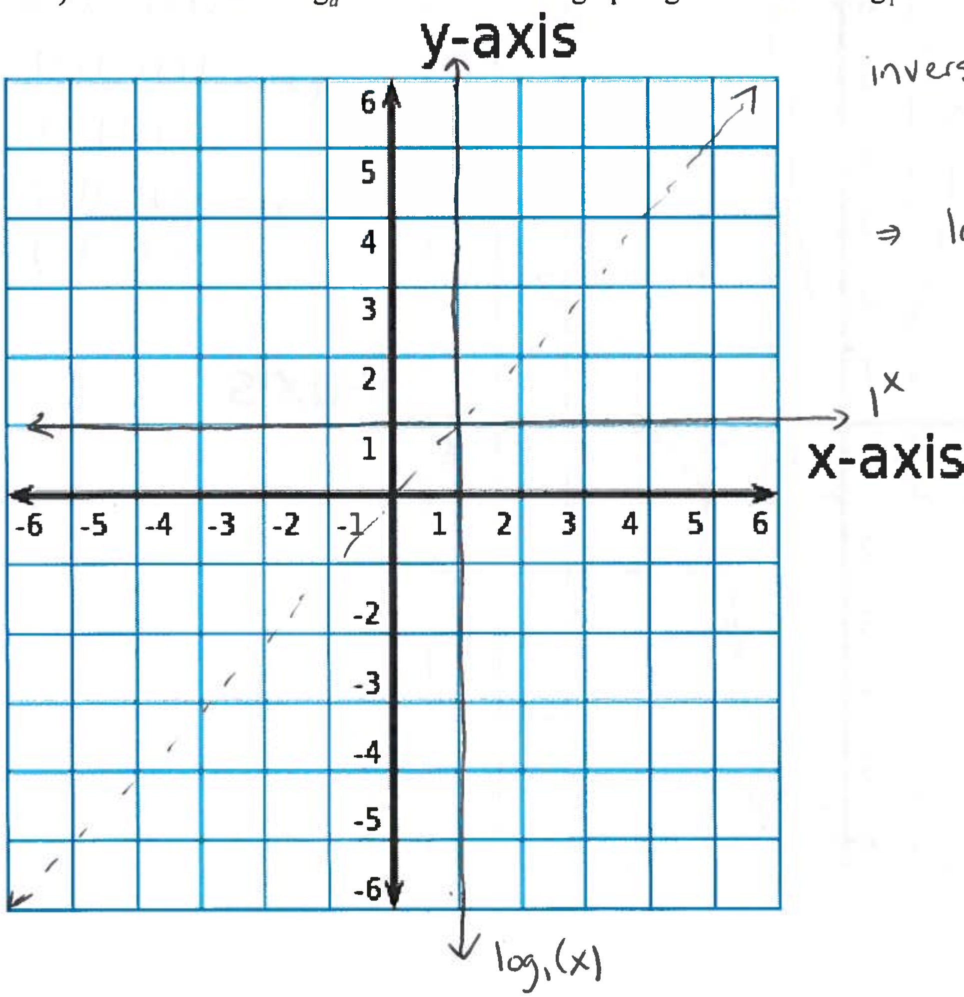
$\log_e(e^{4x-4}) = \log_e(4)$

$4x-4 = \log_e(4)$

$4x = \log_e(4)+4$

$\boxed{x = \frac{\log_e(4)+4}{4}}$

22.) Use that a^x and $\log_a x$ are inverses and graphing to show that $\log_1 x$ is not a function



inverse of $\log_1(x)$
$= 1^x = 1$

$\Rightarrow$ $\log_1(x)$ is a vertical line which fails the vertical line test

$\Rightarrow$ $\log_1(x)$ is not a function

23.) If g(x) is a peicewise defined function with g(x)= $\begin{cases} 2^x & \text{if } x\in(-\infty, 2) \\ -x & \text{if } x\in(3,5] \\ -3 & \text{if } x\in(5,\infty) \end{cases}$

then evaluate the following if able, and if not possible write undefined and state why

a) g(0) $0 \in (-\infty, 2)$

$\Rightarrow g(x) = 2^x \Rightarrow g(0) = 2^0 = \boxed{1}$

b) g(2.5) $2.5 \notin (-\infty, 2)$ or $(3,5]$

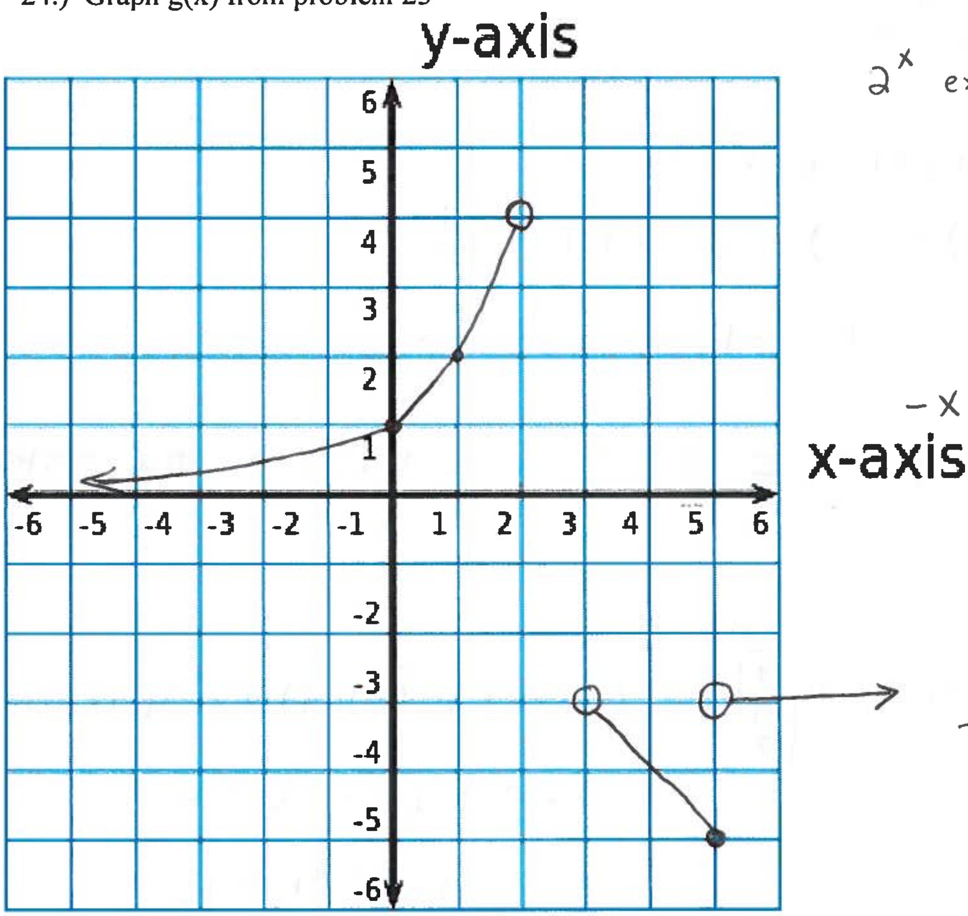
$\Rightarrow g(2.5)$ is $\boxed{\text{undefined}}$ as 2.5 is not in the domain of g

c) g(6)

$6 \in (5, \infty)$

$\Rightarrow g(x) = -3 \Rightarrow g(6) = \boxed{-3}$

24.) Graph g(x) from problem 23



2^x exponential

x	y
0	1
1	$2^1 = 2$
2	$2^2 = 4$

$2 \notin (-\infty, 2)$

$-x$ is linear

x	y
3	-3
4	-4
5	-5

$3 \notin (3, 5]$

-3 is constant

x	y
5	-3
6	-3

$5 \notin (5, \infty)$

25.) Solve the following system of equations for x and y.

$$2x+4y=7$$
$$(3x-2y=-5)\times 2$$

$$\begin{array}{r} 2x+4y=7 \\ +\ 6x-4y=-10 \\ \hline 8x \quad = -3 \end{array}$$

$$\boxed{x=\frac{-3}{8}}$$

$$2\left(\frac{-3}{8}\right)+4y=7$$

$$-\frac{6}{8}+4y=7 \Rightarrow -\frac{3}{4}+4y=7$$

$$4y=7+\frac{3}{4}=\frac{28}{4}+\frac{3}{4}$$

$$4y=\frac{31}{4}$$

$$\boxed{y=\frac{31}{16}}$$

26.) Is $x=1, y=2, z=3$ a solution to the system of three linear equations of three variables

$$x+y+z=6$$
$$2x-y+3z=9$$
$$-x+y-z=-2$$

$(1)+(2)+(3)=6$ ✓

$2(1)+-1(2)+3(3)=2-2+9=9$ ✓

$-(1)+(2)-(3)=-1+2-3=-2$ ✓

$\Rightarrow$ $x=1, y=2, z=3$ is a solution to the system

27.) Find the product of $(1, 3, 2, 0)\begin{pmatrix}-2\\4\\-1\\7\end{pmatrix}$ $=(1)(-2)+(3)(4)+(2)(-1)+(0)(7)$

$=-2+12-2+0$

$=12-4=\boxed{8}$

28.) Find $2\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ $= \begin{pmatrix} 4 \\ -2 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} 4+3 \\ -2+6 \\ 10-12 \end{pmatrix} = \boxed{\begin{pmatrix} 7 \\ 4 \\ -2 \end{pmatrix}}$

29.) Find $\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & 4 \end{bmatrix}$ $= \begin{bmatrix} (1)(3)+(-2)(-2) & (1)(2)+(-2)(4) \\ (3)(3)+(0)(-2) & (3)(2)+(0)(4) \end{bmatrix}$

$$= \begin{bmatrix} 3+4 & 2-8 \\ 9+0 & 6+0 \end{bmatrix} = \boxed{\begin{bmatrix} 7 & -6 \\ 9 & 6 \end{bmatrix}}$$

30.) Find $\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Identity}}\begin{bmatrix} -2 & 1 & 7 \\ 5 & -1 & 4 \\ -2 & 4 & 0 \end{bmatrix}$ $= \boxed{\begin{bmatrix} -2 & 1 & 7 \\ 5 & -1 & 4 \\ -2 & 4 & 0 \end{bmatrix}}$

Any matrix times
the identity gives back the matrix

31.) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

determinant $= 2(1) - (3)(-1) = 2 + 3 = 5$

$\Rightarrow$ inverse $= \dfrac{1}{\det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \boxed{\dfrac{1}{5}\begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

32.) Given that $\begin{bmatrix} 3 & -1 & -1 \\ 8 & 10 & 3 \\ 2 & 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ -2 & 5 & -17 \\ 4 & -11 & 38 \end{bmatrix}$ find solutions to the system of linear equations of 3 variables:

$$\begin{aligned} 3x - y \quad - z &= 3 \\ 8x + 10y + 3z &= 2 \\ 2x + 3y \quad + z &= 1 \end{aligned}$$

Matrix Equation

$$\begin{bmatrix} 3 & -1 & -1 \\ 8 & 10 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ 8 & 10 & 3 \\ 2 & 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 7 \\ -2 & 5 & -17 \\ 4 & -11 & 38 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(3) + (-2)(2) + 7(1) \\ -2(3) + 5(2) + (-17)(1) \\ 4(3) + (-11)(2) + 38(1) \end{bmatrix} = \begin{bmatrix} 3 - 4 + 7 \\ -6 + 10 - 17 \\ 12 - 22 + 38 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -13 \\ 28 \end{bmatrix} \qquad \boxed{\begin{aligned} x &= 6 \\ y &= -13 \\ z &= 28 \end{aligned}}$$