

Final Exam Topics List

The Final Exam will cover material from all chapters in the book, from Sets and Numbers all the way up to Matrix Equations. There will be 32 questions on the exam and there are 34 chapters so there may not be a question directly taken from each chapter. That said all topics on this review list are topics that I feel are important for this course and that may appear on the final exam.

NOTE: Any and all homework questions are fair game to show up on the final exam, you should understand not just how to solve them, but also the techniques you used so that you could solve a similar problem (like a homework problem with different numbers). All questions from the 3 midterms may also appear on the final exam.

Final Exam Breakdown:

There will be 32 questions on the Final Exam.

8 questions from midterm 1 material (Sets and Numbers up to Intro to Graphs)

8 questions from midterm 2 material (Graph Transforms up to Graphing Polynomials)

8 questions from midterm 3 material (Exponential Functions up to Piecewise Defined Functions)

8 questions from material presented after midterm 3 (Linear Equations in Two Variables up to Matrix Equations)

Chapter Breakdowns:

Midterm 1:

Sets and Numbers: Definition of a set, subset, and the 4 special sets (natural numbers, integers, rational numbers and real numbers).

Rules for Numbers: Know how to use the 4 rules for addition and the 4 rules for multiplication (if not necessarily which rule is which). Know the definition of the Distributive Law and how to use it. Understand intervals, how they are sets, and how intervals can be subsets of other intervals.

Solving Simple Equations: Nothing will directly show up from this chapter, but you should be comfortable solving simple equations in the context of other problems that may show up.

Functions: Be comfortable with the definition of a function, the difference of the domain and target of a function and what the domain means in context of the input (i.e. $f(x): \mathbb{N} \rightarrow \mathbb{R} \Rightarrow x$ is a natural number). Know how to identify constant and identity functions, and how to identify when something is not a function.

Sequences: Know the definition of an arithmetic sequence, and how to predict numbers in an arithmetic sequence. Also know the definition of a geometric sequence as well as how to predict numbers in a geometric sequence.

Sums and Series: Know how to sum the first k terms of an arithmetic sequence as well as how to sum an infinite geometric sequence. Note: $-1 < r < 1$ must be true to use the geometric sequence formula.

Counting 1: Know how to count the number of ways to create a set from independent objects (making a sandwich type problems) and how to order objects in a set (arranging letters into words type

problems).

Counting 2: Know the definition of n choose k , how to evaluate it, and when to use it. Be familiar with both definitions of Pascal's Triangle and how to use entries of Pascal's when solving the Binomial Theorem.

More on Functions: Understand Function Composition, Domain, Range, and Implied Domain.

Intro to Graphs: Be comfortable with drawing graphs on the Cartesian plane, as well as the shapes of graphs of constant functions, identity functions and the other graphs shown in class and the book. Know what the vertical line test is and how to use it, what intercepts are, and the concept of little circles vs giant dots and how they pertain to the domain and range of graphs

Midterm 2:

Graph Transformations: “Before”=horizontal and “after”=vertical transformations. Know how to draw the graph of a linear polynomial (like $p(x)=3x-4$) as a sequence of graph transformations.

Inverse Functions: Know that one-to-one means passes the horizontal line test, and that onto means that the functions has the same range and target. Understand that a function being one-to-one and onto means that a function is invertible (has an inverse). Understand the idea of reversal of assignment. i.e that if $f(3) = 6$, then $f^{-1}(6)=3$. If you are given the graph of a function know how to graph the inverse by flipping the original graph over the line $y = x$. Know about the domain and range of an inverse function. Finally know how to find an inverse if you are given the original function.

N-th Roots: Know how to find the implied domain of a n -th root function. Know how to solve inequality and equality equations that have n -th roots in them for x . (i.e solve $\sqrt[4]{2x+7}=4$ for x). Know how to simplify numbers in a n -th root with prime factorization.

Basics of Polynomials: Know what the coefficients, degree, and leading term of a polynomial are. Know how to add, subtract and multiply polynomials together. Know that the leading term of the product is the product of the leading terms for polynomials.

(Polynomial) Division: Know how to do long division of polynomials and how to properly express the answer if there is a remainder left over. You do NOT need to know how to do synthetic division.

Roots and Factors: Know the definition of a root of a polynomial, and that the root of a polynomial is the same as an x -intercept of the polynomial. Understand that if, a , is a root of the polynomial, then $(x-a)$ is a factor of the polynomial, and that $(x-a)$ will divide the polynomial cleanly. Understand that the degree of a product of polynomials is the sum of the degrees of the factors, and that the degree of the polynomial is a limit on the number of roots it has. An n -th degree polynomial has at most n roots.

Constant and Linear Polynomials: Know what constant and linear polynomials are, and how many roots they can have. Know how to find the root of a linear polynomial. Know what slope is and how to graph linear polynomials by using the x and y -intercepts.

Quadratic Polynomials: Know what a quadratic polynomial is, how to use the completing the square formula to rewrite the polynomial. What the discriminant is and how to use it to find how many roots the polynomial has. What the quadratic formula is and how to use it to find the roots. How to graph a quadratic polynomial by using the completed square form.

Factoring Polynomials: Know what completely factored means and what monic means. Know how to factor a quadratic polynomial based off of how many roots it has (and what the roots are). Know how to factor higher order polynomials by guessing roots and using long division to factor $(x-a)$ out of the original polynomial.

Graphing Polynomials: Know how to graph (roughly) polynomials to the far right and far left. For the interior if the roots are known, know how to find the shape of the polynomial for the whole domain.

Midterm 3:

Exponential Functions: Know the algebraic rules for exponential functions that we went over in class (also on pg. 149-150 in your textbook). Know what the base of an exponential function is and how it effects the graph of the function (base greater than 1, less than 1 but greater than 0, and base=1). Understand the notion of e as a base and how you can have irrational numbers as bases. You will need to be able to simplify problems by changing every term to the same base (if possible) and then using the algebraic rules of exponents. You also need to be able to solve exponential equations for x (without using the logarithms from the later chapter).

Rational Functions: Know what a rational function is and how it relates to polynomial function. Know what the implied domain of a rational function is and how to find it. Know what a vertical asymptote is and how to find it, as well as how to find x -intercepts. Know how to find the leading order term of a rational function and how this relates to the behavior of the graph of the function to the “far left and far right”. Finally be able to pull everything together to graph the shape of a rational function.

Logarithms: Know what a logarithm is and how it relates to an exponential (i.e. the inverse of an exponential function with base a is a logarithm with base a). Know how to solve logarithm equations for x , and how to simplify logarithms (like $\log_3(81)=4$). Know the algebraic rules for logarithmic functions (page 175), as well as how to change base .

Exponential and Logarithmic Equations: Know how to solve exponential equations (the 3 step procedure on pg. 182). Know how to solve Logarithmic equations (the 3 step procedure on pg. 184)

Piecewise Defined Functions: Be familiar with what a piecewise defined function looks like, as well as how to graph them. Know what the absolute value function looks like, $|x|$. Know how to write the absolute value function as a piecewise defined function and how to graph it. Know how to use this to solve absolute value equations (like $|x-2|>4$)

Remaining Material:

Linear Equations in Two Variables: Know what a linear equation in 2 variables is and how to represent the equation as a graph of a line. Know what a system of linear equations in two variables is and how the unique solution of the system is the intersection of the 2 lines. Know what it means for there to be no solutions.

Substitution: Know how to solve a system of linear equations in 2 variables by substitution (or elimination).

Linear Equations in Three Variables: Be able to identify a linear equation in three variables and understand that graphically the solution is a plane in 3-d. Know what it means for a system of linear equations in three variables to have a unique solution, infinitely many solutions, or no solutions and graphically how this can happen.

Row and Columns: Know what row and column vectors look like and how they differ from one another. Know how to multiply a column vector on the left by a row vector on the right (inner product).

Vectors and Scalars: Know when you can and how to add vector together as well as how to multiply a vector by a scalar (a real number).

2x2 Matrices: Know what a matrix is as well as how it differs from a vector. Know how to multiply matrices by scalars as well as how to multiply a matrix by a vector. Know what the identity matrix looks like as well as what happens if you multiply a matrix or a vector by the identity matrix. Know how to multiply two matrices together and what an inverse matrix does.

3x3 Matrices: Know how to do everything that you could do for a 2x2 matrix to a 3x3 matrix.

Determinants and Inverse Matrices: Know how to calculate the determinant of a 2x2 and 3x3 matrix. Know how to use the determinant of a 2x2 matrix in order to find the inverse matrix. Know that if the determinant is 0, then there is no inverse matrix.

Matrix Equations: Know how to write a system of equations as a matrix equation and solve the equation by finding the inverse matrix and multiplying it by the solution vector.

Studying Strategy:

- 1) Read through this final review topics list to identify the concepts that your instructor feels are important and the he will therefore be testing you on.
- 2) Study any topics from this list that feel uncomfortable with.
- 3) Study the homework and midterms spending especial time on those topics that you identified as being uncomfortable with
- 4) Do the practice final exam. Try doing it in a test session without notes and time yourself to get an idea of how long it will take you to complete the final exam
- 5) Compare your answers on the practice final with my solutions and then go back and spend additional time studying and doing similar problems to any you may have missed.