

Math 1180:Lab9

Kyle Gaffney

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1 Exponential Distribution

The Exponential Distribution is a continuous version of the Geometric Distribution. As such it gives information about the time to the next event or a waiting time for a particle leaving a cell. To work with an exponential distribution we need to know what the probabilistic rate, λ of the process is.

Suppose that an event happens at a constant rate λ . Then the random variable, T , which is the time until the first (or next) event occurs has probability density function $f(t)$ and cumulative distribution function $F(t)$ where:

$$\begin{aligned}f(t) &= \lambda e^{-\lambda t} \\F(t) &= 1 - e^{-\lambda t}\end{aligned}$$

Finally the probability that an event hasn't happened by time= t is

$$P(t) = e^{-\lambda t}$$

2 Poisson Process

A Poisson Process is a process for which

1. Events occur at a constant probabilistic rate λ
2. Events are independent

2.1 Poisson Distribution

The Poisson Distribution is related to the exponential distribution in the same way that the binomial distribution is related to the geometric distribution. One can in fact think of the Poisson distribution as the limit of infinitely many trials with an infinitesimally unlikely Bernoulli random variables.

The probability that k events happen between time=0 and time= t is

$$Pr(N = k) = p(k; \lambda t) = \frac{e^{-\lambda t} (\lambda t)^k}{k!}$$

Note that $Pr(N=0)$ is the probability that no events have happened by time= t .

$$Pr(N = 0) = p(0, \lambda t) = \frac{e^{-\lambda t} (\lambda t)^0}{0!} = e^{-\lambda t}$$

Which matches what we get for the exponential distribution.

3 Applications

First let us consider a constant birth process.

$$\begin{aligned} \frac{dP}{dt} &= 2 \\ P(0) &= 0 \end{aligned}$$

Describe what the solution $P(t)$ to this continuous deterministic process would be.

Now suppose that births happens randomly, but at a probabilistic rate so that on average there are 2 births per second. Then our probabilistic rate is $\lambda = 2$. Births are independent of one another and happen at a constant probabilistic rate so this is a Poisson Process. As such the time to next birth is governed by an exponential distribution.

Let us simulate this process in R:

```
time=cumsum(rexp(10,rate=2))
Population=seq(1,10)
plot(time,Population)
abline(a=0,b=2)
```

The command `abline` adds a straight line of y-intercept a , and slope b to a plot.

How well did our stochastic simulation match the deterministic solution?

Now rerun the code but run it for 10000 people. How did the global behavior of the stochastic simulation compare?

So all we needed to simulate the problem was to figure out what the probabilistic rate of our stochastic process was. This allows people to generalize what we did for a simple growth process to account for all kinds of complicated stochastic chemical networks.

4 Assignment for the Week

Suppose that I have a cell with 100 molecules in it. The rate at which molecules leave the cell is 1 molecule every 3 seconds.

1. Is this process Poisson? Why or Why not (justify your answer with the definition of a Poisson Process)
2. What would the probabilistic rate for this process be? Is it constant?
3. Create 1 simulation of this process to completion (molecule population in the cell starts at 100 and goes to 0). How long did it take for all the molecules to leave the cell? What is the expected time for all the molecules to leave?

Hint: instead of the birth process like above consider a death process. What would the initial condition be?

4. Now run 10000 trials and find the time of completion for each and average them. In other words average the final time for 10000 trials. How does this compare to the expected value?