

Math 6750 Fall 2009 – Problem Set 2

Due Tuesday September 22, 2009

1) Suppose that T_{ij} and S_{ij} are components of two second order tensors and that $T_{ij} = C_{ijpq}S_{pq}$ for some set of numbers C_{ijpq} . Here, i, j, p, q each run over the integers 1,2,3, and repeated indices imply summation. Show that C_{ijpq} are the components of a fourth order tensor.

Problem 2–15. Derivation of Transport Equation for a Sedimenting Suspension. There are many parallels among momentum, mass, and energy transport because all three are derived from similar conservation laws. In this problem we derive a microscopic balance describing the concentration distribution $\phi(\mathbf{x}, t)$ of a very dilute suspension of small particles suspended in an incompressible fluid undergoing unsteady flow. [Note: $\phi(\mathbf{x}, t)$ is the local volume fraction of particles in the fluid (i.e. volume of particles/volume of fluid) and hence is dimensionless.]

(a) The flux of particles is the sum of that due to convection, that due to settling \mathbf{q}_s (a vector), and that due to diffusion \mathbf{q}_D (also a vector). These fluxes have units of volume per area per time (e.g., velocity). With this in mind, write an integral balance for the conservation of particles for an arbitrary control volume D . Using the divergence theorem, convert all surface integrals to volume integrals, and so obtain a microscopic equation valid at any point in the flow field.

(b) For small particles of radius a , the flux that is due to settling is given by

$$\mathbf{q}_s = \frac{2}{9} \frac{ga^2 \Delta\rho}{\mu} \phi \mathbf{i}_g,$$

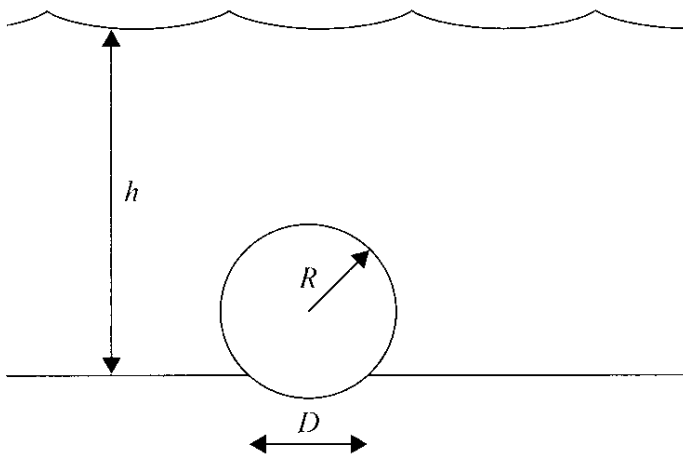
and the diffusive flux by

$$\mathbf{q}_D = -\frac{kT}{6\pi\mu a} \nabla\phi.$$

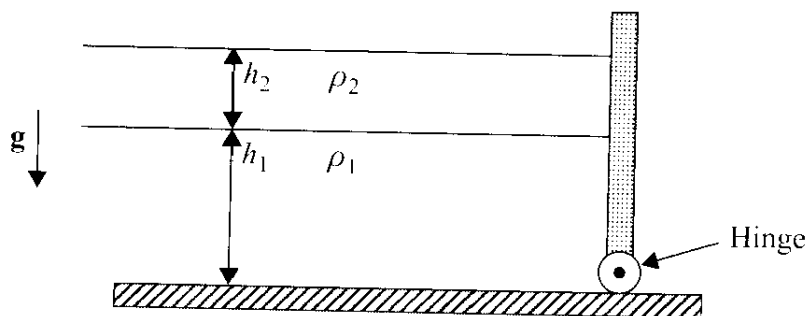
Using these results, obtain an equation in terms of derivatives of the concentration.

(c) Using a characteristic velocity U , characteristic length L , and characteristic time L/U , render the equation dimensionless. What dimensionless parameters appear and what is their physical significance? This equation is the starting point for the study of many problems involving suspensions of particles for which Brownian motion is significant (it is Brownian motion that gives rise to the diffusion term).

Problem 2–17. Fluid Statics. Pool drains can be dangerous things: There was a tragic case recently in this area in which a child was stuck in a drain on the bottom, plugging it, and drowning as a result. Here we look at a somewhat simpler problem. Suppose a ball of radius R is plugging a drain of diameter D at the bottom of a pool of depth h as shown in the figure. Obviously, $R > D/2$ or the ball goes down the drain! Determine the conditions under which the net force on the ball is zero. Assume that the pressure distribution in the drain is just atmospheric pressure, and that in the water is governed by the hydrostatic pressure distribution. If R is 1 ft and D is 6 in, what is the corresponding depth?



Problem 2–20. Fluid Statics. Two fluids are held back by a hinged gate as illustrated in the figure. The lower fluid is of depth h_1 and density ρ_1 and the upper fluid is of depth h_2 and density ρ_2 , with $\rho_2 \leq \rho_1$. Determine the moment per unit width about the base of the hinge. Recall that the moment \mathbf{L} is defined as the cross product of the position vector measured from the point of interest, \mathbf{x} and the force, i.e., $\mathbf{L} = \int \mathbf{r} \wedge \mathbf{f} dS$.



Problem 2–21. Fluid Statics. When a cylindrical tank of liquid is rotated at constant angular velocity ω about its central axis, the free surface attains a steady shape. If the undisturbed (nonrotating) height of the liquid is h , determine the shape of the interface. Be sure to state any and all assumptions. What is the role of interfacial tension? When can it be neglected?