Here is a modification of problem 1 for Math 6610-6620 Problem Set 6
Due January 26, 2009.

1) Consider the Dirichlet problem for the discrete Poisson equation in the unit square $R = [0,1] \times [0.1]$.

$$-(u_{j,l-1} + u_{j-1,l} - 4u_{j,l} + u_{j+1,l} + u_{j,l+1}) = f_{jl} h^2$$

for $j = 1, \ldots, N$ and $l = 1, \ldots, N$ where $(N+1)h = 1$. Assume that the Dirichlet data is homogeneous, that is, $u_{jl} = 0$, for $j = 0$, $j = N+1$, $l = 0$, or $l = N+1$.

Use MATLAB's PCG program (or write your own) to solve this problem using basic CG and preconditioned CG (using MATLAB's CHOLINC or your own incomplete Choleski routine) with different variants of incomplete Choleski factorization as the preconditioner. Use

$$f_{jl} = \sum_{p,q} c_{p,q} \sin\big(p\ j\pi h\big) \cdot \sin\big(q\ l\pi h\big),$$

with

$$c_{p,q} = \frac{1}{p+q}.$$

The corresponding exact solution is

$$u_{jl} = \sum_{p,q} \frac{c_{p,q}}{\lambda^{(p,q)}} \sin(p\ j\pi h) \cdot \sin(q\ l\pi h),$$

where $\lambda^{(p,q)}$ is the eigenvalue of the discrete Laplacian that corresponds to the eigenfunction $\sin(p\ j\pi h) \cdot \sin(q\ l\pi h)$.

For each iterative method, use $u_{jl}^{(0)} = 0$ as initial guess. To compute the error, use the *exact* solution u_{jl} of the *discrete* problem given above. Let $e_{jl}^{(k)} = u_{jl} - u_{jl}^{(k)}$ denote the iteration error after k iterations. For each iterative method, how many iterations k does it take so that $\|e^{(k)}\|_2 \leq \delta \cdot \|e^{(0)}\|_2$ with $\delta = 10^{-4}$? How does the behavior of CG depend on the preconditioner?

Do all of these experiments with $h = .05$ and $h = .025$.

Compare your results with the ones you obtained earlier using Jacobi, Gauss-Seidel, and SOR. You will have to re-run your Jacobi, Gauss-Seidel, and SOR code for the new f specified above.