Mathematical Biology 5120-S09: Problem Set 2

Due Monday February 23.

After trying the problems yourself, I encourage you to talk to each other about how to solve them. If you cannot work a problem out, don't leave it — be sure to come and ask Karin or me for help.

1. Consider one-dimensional diffusion between an absorber at $x = a$ and an absorber at $x = c$. Suppose $a < b < c$ and that a particle is released at $x = b$. If $\mathcal{D}$ is the diffusion coefficient, what is the probability the particle will be absorbed at $x = a$ before it is absorbed at $x = c$? What happens to this probability as $c \to \infty$? How does this result differ from that for diffusion from $r = b$ to a spherical absorber of radius a in an infinite space? If the absorber at $x = a$ is replaced by a reflecting boundary, what is the average number of trips the particle makes from $x = b$ to $x = a$ before reaching $x = c$?

2. Consider the problem for $n(x,t)$: The PDE is $n_t = Dn_{xx}$ for $t > 0$ and $0 < x < L$; the boundary conditions (BC) are $n(0,t) = 1$ and $n_x(L,t) = 0$ for $t > 0$; and the initial condition (IC) is $n(x,0) = 0$ for $0 < x < L$. The inhomogeneous (nonzero) BC at $x = 0$ prevents direct application of the method of separation of variables. We can still solve the problem in two stages as follows. First look for a *steady-state* solution $n^S(x)$ which satisfies $n^S_{xx} = 0$ for $0 < x < L$, with BC $n^S(0) = 1$ and $n^S_x(L) = 0$. Then write the solution to the original problem $n(x,t)$ as the sum of the steady-state solution $n^S(x)$ and an unknown transient function $n^T(x,t)$, that is $n(x,t) = n^S(x) + n^T(x,t)$. By substituting this into the PDE, BC, and IC of the original problem and using information about the function $n^S(x)$, you can derive a PDE, BC, and IC for the function $n^T(x,t)$ and this problem has homogeneous (zero) BC. This problem can be solved by separation of variables and the result added to n^S to get the solution to the original problem.

   a) Solve for the steady-state solution $n^S(x)$.

   b) What PDE, BC, and IC does the function $n^T(x,t)$ satisfy?

   c) Solve the problem for $n^T(x,t)$ by separation of variables.

   d) What happens to $n^T(x,t)$ as $t \to \infty$?

   e) For $D = 1$ and $L = 1$, use Maple or some other program to plot the solution $n(x,t)$ as a function of x for each of $t = 0.25$, $t = 0.50$, $t = 1.0$, $t = 2.0$, and $t = 4.0$. Your solution will involve an infinite sum; truncate this sum after 10 terms before plotting the solution.

Be sure to show all details on each problem for full credit.