Mathematical Biology 5120-S07: Problem Set 1

Due Thursday February 8, before class.

After trying the problems yourself, I encourage you to talk to each other about how to solve them. If you cannot work a problem out, don't leave it — be sure to come and ask for help.

1. Particles can move because of other reasons than diffusion. Suppose particles with number density $N(x,t)$ are suspended in a fluid which is moving with velocity $u(x,t)$ and that the particles move with the fluid ($u(x,t) > 0$ means motion to the right). What is the flux function $J(x,t)$ in this situation (assume the particles do not also diffuse)? What is the partial differential equation (PDE) satisfied by the function $N(x,t)$? What are the flux function and PDE in the case that the particles move both because they are carried by the fluid and because they diffuse?

2. Suppose particles move just by diffusion. Suppose that a number N_0 particles are injected at $x = 0$ at time $t = 0$, and another N_1 particles are injected at the origin at time $t = t_1 > 0$. Write down the function $N(x,t)$ that gives the number density of particles at location x at time t. (Hint: The PDE is linear so you can add solutions. Also note that you will have different formulas for $0 < t < t_1$ and for $t_1 < t$.). Let $t_1 = 1$, and use Maple or some other graphics program to plot the solution at $x = 1$ as a function of time for diffusion coefficients $D = 10^{-3}$ and $D = 10^{-6}$. Comment on the concentration plots you obtain. Now assume instead that particles are injected at rate r per unit time starting at time $t = 0$ (so $r\ dt$ particles are injected in a short time dt). Write down the function $N(x,t)$ as an integral involving the Gaussian solution for diffusion we obtained in class.

Be sure to show all details on each problem for full credit