- Huxley crossbridge model for $v < 0$. (1)

$n_t - v n_x = f(1-n) - gn$

Assume $v<0$ constant and $n_t = 0$.

$f(x) = \frac{f_1 x}{h}$ for $0 \le x \le h$, 0 otherwise

$g(x) = \frac{g_1 x}{h}$ for $0 \le x$, g_2 otherwise

$$-vn' = f(1-n) - gn$$

Consider $x > h$.

$$-vn' = \frac{-g_1 x}{h} n \Rightarrow \frac{n'}{n} = \frac{g_1}{vh} x$$

$$\ln n(x) - \ln n(h) = \int_h^x \frac{g_1}{vh} s\, ds = \frac{g_1}{2vh}(x^2 - h^2)$$

$$\Rightarrow n(x) = n(h)\, e^{\frac{g_1}{2vh}(x^2-h^2)} \quad \text{for } x \ge h.$$

Consider $0 \le x \le h$.

$$-vn' = \frac{f_1 x}{h}(1-n) - \frac{g_1 x}{h} = \frac{f_1 x}{h} - \frac{(f_1+g_1)}{h} x n$$

$$\Rightarrow n' = -\frac{f_1 x}{vh} + \frac{(f_1+g_1)}{vh} x n$$

$$\Rightarrow n' - \left(\frac{f_1+g_1}{vh}\right) x n = -\frac{f_1}{vh} x$$

$$\frac{d}{dx}\left(n e^{-\frac{1}{2}\left(\frac{f_1+g_1}{vh}\right)x^2}\right) = -\frac{f_1}{vh} x e^{-\frac{1}{2}\left(\frac{f_1+g_1}{vh}\right)x^2}$$

$$n(x) e^{-\frac{(f_1+g_1)}{2vh}x^2} - n(0) = \frac{f_1}{vh}\int_0^x (-s) e^{-\frac{(f_1+g_1)}{2vh}s^2} ds$$

$$= \left(\frac{f_1}{vh}\right)\left(\frac{vh}{f_1+g_1}\right)\left[e^{-\frac{(f_1+g_1)}{2vh}x^2} - 1\right]$$

Since $n(0) = 0$ we have.

$$n(x) = \frac{f_1}{f_1 + g_1} \left\{ 1 - e^{\frac{(f_1+g_1)}{2vh} x^2} \right\} \quad \text{for} \quad 0 \le x \le h.$$

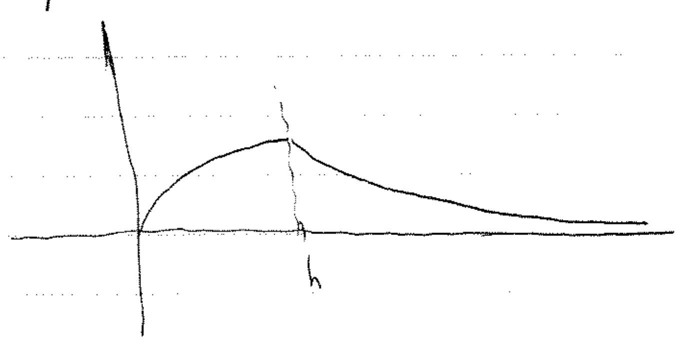
$$n(h) = \frac{f_1}{f_1+g_1} \left(1 - e^{\frac{(f_1+g_1)h}{2v}} \right)$$

Let $\phi \equiv (f_1+g_1)h/2$. Then

$$n(h) = \frac{f_1}{f_1+g_1} \left(1 - e^{\phi/v} \right)$$

So

$$n(x) = \begin{cases} \dfrac{f_1}{f_1+g_1} \left(1 - e^{\frac{\phi}{v}\left(\frac{x}{h}\right)^2} \right), & 0 \le x \le h. \\[2ex] \dfrac{f_1}{f_1+g_1} \left(1 - e^{\frac{\phi}{v}} \right) e^{\frac{g_1}{2vh}(x^2 - h^2)}, & x \ge h. \end{cases}$$



$$P = c\rho \int_{-\infty}^{\infty} k x \, n(x) \, dx$$

$$= c\rho k \left\{ \int_0^h \frac{f_1}{f_1+g_1} \, x \left(1 - e^{\frac{\phi}{v}\left(\frac{x}{h}\right)^2}\right) dx \right.$$

$$\left. + \int_h^\infty \frac{f_1}{f_1+g_1} \left(1 - e^{\phi/v}\right) e^{\frac{g_1}{2vh}(x^2-h^2)} \, x \, dx \right\}$$

$$= \left(c\rho k \, \frac{f_1}{f_1+g_1}\right) \left(\int_0^h x \left(1 - e^{\frac{\phi}{v}\left(\frac{x}{h}\right)^2}\right) dx \right.$$

$$\left. + \left(1 - e^{\phi/v}\right) \int_h^\infty x \, e^{\frac{g_1}{2vh}(x^2-h^2)} \, dx \right)$$

$$I_1 \equiv \int_0^h x \left(1 - e^{\frac{\phi}{v}\left(\frac{x}{h}\right)^2}\right) dx = \frac{h^2}{2} - \int_0^h x \, e^{\frac{\phi}{v}\left(\frac{x}{h}\right)^2} dx$$

$$= \frac{h^2}{2} - \frac{vh^2}{2\phi} \, e^{\frac{\phi}{v}\left(\frac{x}{h}\right)^2} \Big|_0^h$$

$$= \frac{h^2}{2} \left\{ 1 - \frac{v}{\phi} \left(e^{\phi/v} - 1 \right) \right\}$$

$$I_2 = \int_h^\infty x \, e^{\frac{g_1}{2vh}(x^2-h^2)} \, dx = \frac{2vh}{g_1 2} \, e^{\frac{g_1}{2vh}(x^2-h^2)} \Big|_h^\infty$$

$$= \frac{vh}{g_1} \left(0 - e^0 \right) = -\frac{vh}{g_1}$$

So
$$P = c\rho k \frac{f_1}{f_1+g_1}\left\{\frac{h^2}{2}\left(1 - \frac{v}{\phi}e^{\phi/v} + \frac{v}{\phi}\right) - \frac{vh}{g_1}\left(1 - e^{\phi/v}\right)\right\}$$

$$= c\rho k \frac{f_1}{f_1+g_1}\,\frac{h^2}{2}\left\{1 - \frac{v}{\phi}e^{\phi/v} + \frac{v}{\phi} - \frac{2v}{g_1 h}\left(1 - e^{\phi/v}\right)\right\}$$

$$= P_0\left\{1 + v\left(\frac{1}{\phi} - \frac{2}{g_1 h}\right) + v e^{\phi/v}\left(\frac{2}{g_1 h} - \frac{1}{\phi}\right)\right\}$$

$$\frac{2}{g_1 h} - \frac{1}{\phi} = \frac{2}{g_1 h} - \frac{2}{h(f_1+g_1)} = \frac{2}{h}\left\{\frac{1}{g_1} - \frac{1}{f_1+g_1}\right\}$$
$$= \frac{2}{h}\,\frac{f_1}{g_1(f_1+g_1)} = \frac{f_1}{g_1\phi}$$

$$P = P_0\left\{1 + \left(-\frac{f_1}{g_1\phi}\right)v + v e^{\phi/v}\frac{f_1}{g_1\phi}\right\}$$

$$= P_0\left\{1 - \frac{f_1}{g_1}\frac{v}{\phi}\left(1 - e^{\phi/v}\right)\right\}$$

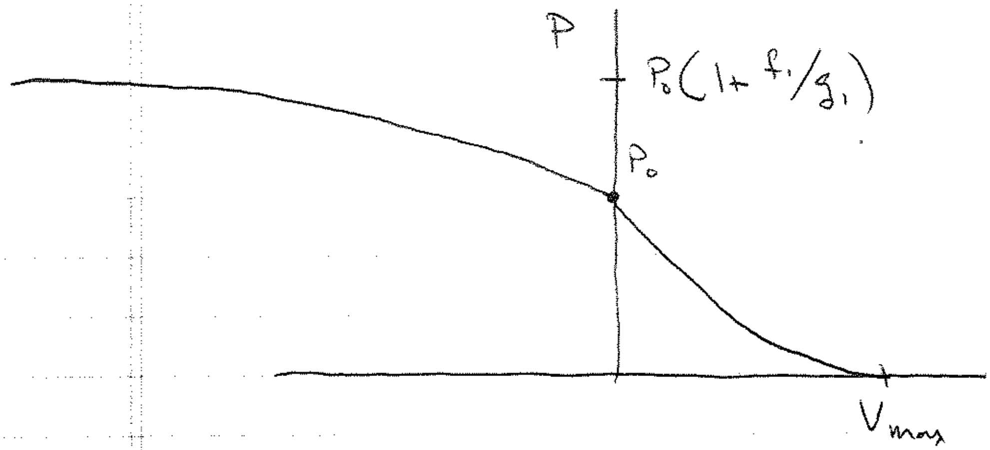
What happens to P as $v \to -\infty$.

$$e^{\phi/v} = 1 + \phi/v + \tfrac{1}{2}(\phi/v)^2 + \cdots$$

$$1 - e^{\phi/v} = -\frac{\phi}{v} - \frac{1}{2}\left(\frac{\phi}{v}\right)^2 - \cdots$$

so
$$P = P_0\left\{1 - \frac{f_1}{g_1}\frac{v}{\phi}\left(-\frac{\phi}{v} - \frac{1}{2}\left(\frac{\phi}{v}\right)^2 + O\left(\left(\frac{\phi}{v}\right)^3\right)\right)\right\}$$

$$= P_0\left\{1 + \frac{f_1}{g_1} + O\left(\frac{\phi}{v}\right)\right\} \to P_0\left(1 + \frac{f_1}{g_1}\right)$$



$$P(v) = P_0 \begin{cases} 1 - \dfrac{v}{\phi}\left(1 - e^{-\phi/v}\right)\left(1 + \dfrac{1}{2}\left(\dfrac{f_1+g_1}{g_2}\right)^2 \dfrac{v}{\phi}\right), & v > 0 \\[2ex] 1 - \dfrac{f_1}{g_1}\,\dfrac{v}{\phi}\left(1 - e^{\phi/v}\right) & v < 0 \end{cases}$$

$P(v)$ is C^0 but not necessarily C^1 at $v=0$.