$S_t \circ c(h)(a_s)_{ti} c(s) + \mathfrak{S}_e m^i n(a_r)$

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On pathwise limit theorems

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Assume that $Z_1, Z_2, \ldots$ are real random variables, which converge weakly to some limit. In this talk we pursue the question of the pathwise behavior of the sequence (Z_k). For example, let $Z_k = (X_1 + X_2 + \cdots + X_k)/s_k$, where $X_1, X_2, \ldots$ are independent with $EX_k = 0$ and $\mathrm{Var}(X_1 + \cdots + X_k) = s_k^2$. Under mild assumptions we get by the central limit theorem that Z_k is asymptotically standard normal. However, the pathwise behavior of such processes is different. If the X_k are identically distributed we get by Lévy's classical arc–sine law that

$$\frac{1}{n} \sum_{k=1}^{n} I\{Z_k \le x\} \tag{1}$$

converges weakly to the arc–sine distribution if $x = 0$. On the other hand, if e.g. $s_k^2 = \exp((\log k)^{1+\varepsilon})$ $(\varepsilon > 0)$, we show that (1) converges almost surely to $\Phi(x)$, the standard normal distribution function. By considering weighted averages

$$\frac{1}{D_n} \sum_{k=1}^{n} d_k I\{Z_k \le x\}, \quad d_k \ge 0,\ D_n = \sum_{k=1}^{n} d_k, \tag{2}$$

instead of the ordinary mean in (1), we can also obtain the almost sure convergence to $\Phi(x)$ in the i.i.d. case. This is the so–called *almost sure central limit theorem*. The main purpose of the talk is to study the pathwise behavior of weakly convergent sequences under the aspect of different averaging methods in (2).