The credit rating of a firm describes its credit quality and the rating migration process has direct impact on the value of the firm and its ability to raise capital. It could also have far reaching effects on the financial market where various derivatives may have been written on it. In practice, a Markov chain model is assumed and the probability transition matrix is estimated from historical data. We use a jump diffusion process and several barriers to describe the Markov chain, and it leads to a partial integro-differential equation (PIDE) formulation, with defaults and rating transitions characterized by barrier crossings. Efficient and reliable numerical solutions are developed for the variable coefficient equation and proper choices of the coefficients generate a transition matrix that is in good agreement with historical data, across all credit ratings. A simple adjustment in the credit index drift converts the model to be used in a risk-neutral setting, which makes it a valuable tool in credit derivative pricing.