$S_t \circ c(h)(a_s)_{ti}c(s) + \mathfrak{S}_e m^i n(a_r)$

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Sparsity recovery in the high-dimensional and noisy setting: Practical and information-theoretic limitations

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The problem of recovering the sparsity pattern of an unknown signal arises in various areas of applied mathematics and statistics, including constructive approximation, compressive sensing, and model selection. The standard optimization-theoretic formulation of sparsity recovery involves ℓ_0-constraints, and typically leads to computationally intractable problems. This difficulty motivates the development and analysis of approximate methods; in particular, a great deal of work over the past decade has focused on the use of ℓ_1-relaxations and related convex methods for sparsity recovery. We consider the high-dimensional and noisy setting, in which one makes n noisy observations of an unknown signal in p dimensions with at most s non-zero entries. Of interest is the number of observations n that are required, as a function of the model dimension p and sparsity index s, to correctly estimate the support of the signal. For a broad class of random Gaussian measurement ensembles, we provide sharp upper and lower bounds on the performance of a computationally efficient method (ℓ_1-constrained quadratic programming), as well as information-theoretic upper and lower bounds on the performance of any method (regardless of its computational efficiency). We discuss connections to other work, and some open problems in this rapidly-growing field.

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[0] Jointly with the Applied Math Seminar