$S_t \circ c(h)(a_s)_{ti}c(s) + \mathfrak{S}_e m^i n(a_r)$

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Uniform Limit Theorems for Wavelet Density Estimators

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(This is joint work with Evarist Giné). If $X_1, ..., X_n$ is an i.i.d. sample from the density p_0 on the real line, one can estimate p_0 by a (truncated) wavelet expansion p_n of the empirical measure. We will discuss several probabilistic limit theorems for p_n: First, the almost sure rate of convergence of $\sup_{y\in\mathbb{R}} |p_n(y) - Ep_n(y)|$ is established, as well as an exact law of the logarithm for a suitably scaled version of this quantity. This implies that $\sup_{y\in\mathbb{R}} |p_n(y) - p_0(y)|$ attains the optimal almost sure rate of convergence for estimating p_0, if j_n is suitably chosen. Second, a uniform central limit theorem as well as strong invariance principles for the distribution function of p_n, that is, for the stochastic processes $\sqrt{n}\int_{-\infty}^{s}(p_n - p_0), s \in \mathbb{R}$, are obtained; and more generally, uniform central limit theorems for the processes $\sqrt{n}\int(p_n - p_0)f; f \in \mathcal{F}$, for other Donsker classes $\mathcal{F}$. The mathematical ingredients of the proofs – next to wavelet theory mostly empirical process theory, in particular, Talagrand's inequality and suitable expectation inequalities – are discussed. We also discuss statistical applications, in particular, it is shown that essentially the same limit theorems can be obtained for the hard thresholding wavelet estimator introduced by Donoho, Johnstone, Kerkyacharian and Picard (1996).