Drunken Sailors in Disordered Cities

Firas Rassoul-Agha

Department of Mathematics University of Utah

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- The object is Brownian motion: a random path
- Has many applications
- Will reveal unexpected connections between seemingly different phenomena



























































Looks like stock prices!!

Looks like stock prices!! Will also see link to thermodynamics!





Random Path



Random Path



Random Path



This is Brownian motion

This is Brownian motion

• The eagle's eye view of the drunken sailor's path

This is Brownian motion

- The eagle's eye view of the drunken sailor's path
- It is a random path

Physics of Brownian Motion

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Lucretius (60 BC), On the Nature of Things!!!

Physics of Brownian Motion



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Dutch physiologist Jan Ingenhousz (1785):

coal dust particles on surface of alcohol.


Physics of Brownian Motion



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Physics of Brownian Motion



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German physicist Albert Einstein (1905) and Polish physicist Marian Smoluchowski (1906): brought the object to the attention of physicists. Indirectly confirmed the existence of atoms and molecules.



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Japanese mathematician Kiyoshi Itô (1942-1946): derived a calculus – and thereby a theory of stochastic differential equations – that is completely different from the classical theory.

American mathematician Monroe Donsker (1951): proved connection to drunken sailors.



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and more ...

Next 1: complex landscape (disordered city)







































Accounts for inhomogeneity (particle-environment interaction)

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Applications:

Accounts for inhomogeneity (particle-environment interaction)

Applications:

• propagation of radioactivity in a rocky landscape

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Accounts for inhomogeneity (particle-environment interaction)

Applications:

- propagation of radioactivity in a rocky landscape
- chromatin motion (trying to bind to a site on the <u>complex</u> DNA) and more...

Next 2: other drunken sailors









or cars...





















Firas Rassoul-Agha, University of Utah

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Drunken Sailors in Disordered Cities

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Particle-Particle Interaction

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• traffic models (actually used in Rome!)

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"Everything should be made as simple as possible, but not simpler."

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