Particle models with interaction through the center of mass

János Engländer

University of Colorado at Boulder

Frontier Probability Days
Salt Lake City, March 10-12, 2011

In this talk,

C.O.M.: Center of mass

Model

Branching Brownian motion with interaction:

Model

Branching Brownian motion with interaction:

- Motion: Brownian motion on $\mathbb{R}^d$.

Model

Branching Brownian motion with interaction:

- Motion: Brownian motion on $\mathbb{R}^d$.
- Branching: dyadic (2 offspring), unit time.

Model

Branching Brownian motion with interaction:

- Motion: Brownian motion on $\mathbb{R}^d$.
- Branching: dyadic (2 offspring), unit time.
- Start with single particle at the origin.

Model

Branching Brownian motion with interaction:

- Motion: Brownian motion on $\mathbb{R}^d$.
- Branching: dyadic (2 offspring), unit time.
- Start with single particle at the origin.
- Let $\gamma \neq 0$. If Z denotes the process and Z_t^i is the i^{th} particle, then Z_t^i 'feels' the drift

$$\frac{\gamma}{n_t} \sum_{1 \le j \le n_t} Z_t^j - \cdot$$

Model

Branching Brownian motion with interaction:

- Motion: Brownian motion on $\mathbb{R}^d$.
- Branching: dyadic (2 offspring), unit time.
- Start with single particle at the origin.
- Let $\gamma \neq 0$. If Z denotes the process and Z_t^i is the i^{th} particle, then Z_t^i 'feels' the drift

$$\frac{\gamma}{n_t} \sum_{1 \leq j \leq n_t} Z_t^j - \cdot$$

That is the particle's infinitesimal generator is

$$\boxed{\tfrac{1}{2}\Delta + \tfrac{\gamma}{n_t} \textstyle\sum_{1 \leq j \leq n_t} \left(Z_t^j - x\right) \cdot \nabla.}$$

($n_t := 2^{\lfloor t \rfloor}$, where $\lfloor t \rfloor$ is the integer part of t.)

Model

Branching Brownian motion with interaction:

- Motion: Brownian motion on $\mathbb{R}^d$.
- Branching: dyadic (2 offspring), unit time.
- Start with single particle at the origin.
- Let $\gamma \neq 0$. If Z denotes the process and Z_t^i is the i^{th} particle, then Z_t^i ‘feels’ the drift
$$\frac{\gamma}{n_t} \sum_{1 \le j \le n_t} Z_t^j - \cdot$$
That is the particle’s infinitesimal generator is
$$\boxed{\tfrac{1}{2}\Delta + \tfrac{\gamma}{n_t} \textstyle\sum_{1 \le j \le n_t} \left(Z_t^j - x\right) \cdot \nabla.}$$
($n_t := 2^{\lfloor t \rfloor}$, where $\lfloor t \rfloor$ is the integer part of t.)
If $\gamma > 0$: attraction; if $\gamma < 0$: repulsion.

Some people working on similar models

- **Simon Harris** (Bath)

Some people working on similar models

- **Simon Harris** (Bath)
- **Julien Berestycki** (Paris)

Some people working on similar models

- **Simon Harris** (Bath)
- **Julien Berestycki** (Paris)
- **Omer Adelman** (Paris)

Some people working on similar models

- **Simon Harris** (Bath)
- **Julien Berestycki** (Paris)
- **Omer Adelman** (Paris)
- **Jin Feng** (Kansas) — new project: take $g : \mathbb{R}_+ \to \mathbb{R}$ and

$$\frac{1}{n_t} \sum_{1 \le j \le n_t} g\left(|Z_t^j - \cdot|\right) \frac{Z_t^j - \cdot}{|Z_t^j - \cdot|},$$

instead of just $g(y) = \gamma y$.

Some people working on similar models

- **Simon Harris** (Bath)
- **Julien Berestycki** (Paris)
- **Omer Adelman** (Paris)
- **Jin Feng** (Kansas) — new project: take $g : \mathbb{R}_+ \to \mathbb{R}$ and

$$\frac{1}{n_t} \sum_{1 \le j \le n_t} g\left(|Z_t^j - \cdot|\right) \frac{Z_t^j - \cdot}{|Z_t^j - \cdot|},$$

  instead of just $g(y) = \gamma y$.
- **H. Gill** (U. British Columbia): super-Brownian motion with self-interaction

Some people working on similar models

- **Simon Harris** (Bath)
- **Julien Berestycki** (Paris)
- **Omer Adelman** (Paris)
- **Jin Feng** (Kansas) — new project: take $g : \mathbb{R}_+ \to \mathbb{R}$ and

$$\frac{1}{n_t} \sum_{1 \le j \le n_t} g\left(|Z_t^j - \cdot|\right) \frac{Z_t^j - \cdot}{|Z_t^j - \cdot|},$$

  instead of just $g(y) = \gamma y$.
- **H. Gill** (U. British Columbia): super-Brownian motion with self-interaction
- **M. Balázs**, **B. Tóth** and **M. Rácz** (Budapest Technical U.): A particle system interacting through the C.O.M.

C.O.M. stabilizes

Let $\overline{Z}_t := \frac{1}{n_t} \sum_{i=1}^{n_t} Z_t^i$, that is, $\overline{Z}$ is the **C.O.M.** for Z.

C.O.M. stabilizes

Let $\overline{Z}_t := \frac{1}{n_t}\sum_{i=1}^{n_t} Z_t^i$, that is, $\overline{Z}$ is the **C.O.M.** for Z. Notice that

$$\frac{1}{n_t}\sum_{1\leq j\leq n_t}\left(Z_t^j - Z_t^i\right) = \overline{Z}_t - Z_t^i, \tag{1}$$

and so the net attraction pulls the particle towards the C.O.M. (net repulsion pushes it away from the C.O.M.).

C.O.M. stabilizes

Let $\overline{Z}_t := \frac{1}{n_t}\sum_{i=1}^{n_t} Z_t^i$, that is, $\overline{Z}$ is the **C.O.M.** for Z. Notice that

$$\frac{1}{n_t}\sum_{1\le j\le n_t}\left(Z_t^j - Z_t^i\right) = \overline{Z}_t - Z_t^i, \tag{1}$$

and so the net attraction pulls the particle towards the C.O.M. (net repulsion pushes it away from the C.O.M.).

Lemma (C.O.M. stabilizes)

$\lim_{t\to\infty}\overline{Z}_t = N$, *a.s.* *where* $N \sim \mathcal{N}(0, 2\mathbf{I}_d)$.

C.O.M. stabilizes

Let $\overline{Z}_t := \frac{1}{n_t}\sum_{i=1}^{n_t} Z_t^i$, that is, $\overline{Z}$ is the **C.O.M.** for Z. Notice that

$$\frac{1}{n_t}\sum_{1\le j\le n_t}\left(Z_t^j - Z_t^i\right) = \overline{Z}_t - Z_t^i, \tag{1}$$

and so the net attraction pulls the particle towards the C.O.M. (net repulsion pushes it away from the C.O.M.).

Lemma (C.O.M. stabilizes)

$\lim_{t\to\infty}\overline{Z}_t = N$, *a.s.* *where* $N \sim \mathcal{N}(0, 2\mathbf{I}_d)$.

Proof: Elementary proof, using independence and Brownian scaling. □

C.O.M. stabilizes

Let $\overline{Z}_t := \frac{1}{n_t}\sum_{i=1}^{n_t} Z_t^i$, that is, $\overline{Z}$ is the **C.O.M.** for Z. Notice that

$$\frac{1}{n_t}\sum_{1\le j\le n_t}\left(Z_t^j - Z_t^i\right) = \overline{Z}_t - Z_t^i, \tag{1}$$

and so the net attraction pulls the particle towards the C.O.M. (net repulsion pushes it away from the C.O.M.).

Lemma (C.O.M. stabilizes)

$\lim_{t\to\infty}\overline{Z}_t = N$, *a.s.* *where* $N \sim \mathcal{N}(0, 2\mathbf{I}_d)$.

Proof: Elementary proof, using independence and Brownian scaling. □

Remark: In fact, $\overline{Z}$ *is a Markov process* w.r. to canonical filtration for Z.

Viewing the system from C.O.M.

Assume that $t \in [m, m+1)$. When viewed from $\overline{Z}$, the relocation of a particle is governed by

$$\mathrm{d}(Z_t^1 - \overline{Z}_t) = \mathrm{d}Z_t^1 - \mathrm{d}\overline{Z}_t = \mathrm{d}B_t^{m,1} - 2^{-m} \sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma(Z_t^1 - \overline{Z}_t)\mathrm{d}t.$$

Viewing the system from C.O.M.

Assume that $t \in [m, m+1)$. When viewed from $\overline{Z}$, the relocation of a particle is governed by

$$\mathrm{d}(Z_t^1 - \overline{Z}_t) = \mathrm{d}Z_t^1 - \mathrm{d}\overline{Z}_t = \mathrm{d}B_t^{m,1} - 2^{-m} \sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma(Z_t^1 - \overline{Z}_t)\mathrm{d}t.$$

So if $Y^1 := Z^1 - \overline{Z}$, then

$$\mathrm{d}Y_t^1 = \mathrm{d}B_t^{m,1} - 2^{-m} \sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma Y_t^1 \mathrm{d}t.$$

Viewing the system from C.O.M.

Assume that $t \in [m, m+1)$. When viewed from $\overline{Z}$, the relocation of a particle is governed by

$$\mathrm{d}(Z_t^1 - \overline{Z}_t) = \mathrm{d}Z_t^1 - \mathrm{d}\overline{Z}_t = \mathrm{d}B_t^{m,1} - 2^{-m}\sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma(Z_t^1 - \overline{Z}_t)\mathrm{d}t.$$

So if $Y^1 := Z^1 - \overline{Z}$, then

$$\mathrm{d}Y_t^1 = \mathrm{d}B_t^{m,1} - 2^{-m}\sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma Y_t^1 \mathrm{d}t.$$

Clearly, letting $\tau := t - \lfloor t \rfloor$, one has

$$B_\tau^{m,1} - 2^{-m}\bigoplus_{i=1}^{2^m} B_\tau^{m,i} = -\bigoplus_{i=2}^{2^m} 2^{-m} B_\tau^{m,i} \oplus (1 - 2^{-m}) B_\tau^{m,1}.$$

Viewing the system from C.O.M.

Assume that $t \in [m, m+1)$. When viewed from $\overline{Z}$, the relocation of a particle is governed by

$$\mathrm{d}(Z_t^1 - \overline{Z}_t) = \mathrm{d}Z_t^1 - \mathrm{d}\overline{Z}_t = \mathrm{d}B_t^{m,1} - 2^{-m} \sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma(Z_t^1 - \overline{Z}_t)\mathrm{d}t.$$

So if $Y^1 := Z^1 - \overline{Z}$, then

$$\mathrm{d}Y_t^1 = \mathrm{d}B_t^{m,1} - 2^{-m} \sum_{i=1}^{2^m} \mathrm{d}B_t^{m,i} - \gamma Y_t^1 \mathrm{d}t.$$

Clearly, letting $\tau := t - \lfloor t \rfloor$, one has

$$B_\tau^{m,1} - 2^{-m} \bigoplus_{i=1}^{2^m} B_\tau^{m,i} = -\bigoplus_{i=2}^{2^m} 2^{-m} B_\tau^{m,i} \oplus (1 - 2^{-m}) B_\tau^{m,1}.$$

The RHS is a Brownian motion with mean zero and variance $(1 - 2^{-m})\tau \mathbf{I}_d := \sigma_m^2 \tau \mathbf{I}_d$.

That is,

$$dY_t^1 = \sigma_m \, dW^1(t) - \gamma Y_t^1 dt,$$

where W^1 is a standard Brownian motion.

That is,

$$\mathrm{d}Y_t^1 = \sigma_m \,\mathrm{d}W^1(t) - \gamma Y_t^1 \mathrm{d}t,$$

where W^1 is a standard Brownian motion.
Hence, on $[m, m+1)$, the relocation viewed from the C.O.M. is governed by the O-U generator:

$$\boxed{\tfrac{1}{2}\sigma_m \,\Delta - \gamma x \cdot \nabla \approx \tfrac{1}{2}\Delta - \gamma x \cdot \nabla.}$$

That is,

$$\mathrm{d}Y_t^1 = \sigma_m \,\mathrm{d}W^1(t) - \gamma Y_t^1 \mathrm{d}t,$$

where W^1 is a standard Brownian motion.
Hence, on $[m, m+1)$, the relocation viewed from the C.O.M. is governed by the O-U generator:

$$\boxed{\tfrac{1}{2}\sigma_m \,\Delta - \gamma x \cdot \nabla \approx \tfrac{1}{2}\Delta - \gamma x \cdot \nabla.}$$

:(Independence of particles: lost

That is,

$$\mathrm{d}Y_t^1 = \sigma_m \,\mathrm{d}W^1(t) - \gamma Y_t^1 \mathrm{d}t,$$

where W^1 is a standard Brownian motion.
Hence, on $[m, m+1)$, the relocation viewed from the C.O.M. is governed by the O-U generator:

$$\boxed{\tfrac{1}{2}\sigma_m \,\Delta - \gamma x \cdot \nabla \approx \tfrac{1}{2}\Delta - \gamma x \cdot \nabla.}$$

:(Independence of particles: lost

:) But easy to show asymptotically vanishing correlation between driving BM's and that their "degree of freedom" is $2^m - 1$.

That is,

$$\mathrm{d}Y_t^1 = \sigma_m \,\mathrm{d}W^1(t) - \gamma Y_t^1 \mathrm{d}t,$$

where W^1 is a standard Brownian motion.
Hence, on $[m, m+1)$, the relocation viewed from the C.O.M. is governed by the O-U generator:

$$\boxed{\tfrac{1}{2}\sigma_m \,\Delta - \gamma x \cdot \nabla \approx \tfrac{1}{2}\Delta - \gamma x \cdot \nabla.}$$

:(Independence of particles: lost

:) But easy to show asymptotically vanishing correlation between driving BM's and that their "degree of freedom" is $2^m - 1$.

Q: How can we put together that $\overline{Z}_t$ tends to a random final position a.s. with the description of the system 'as viewed from $\overline{Z}_t$?'

Q: How can we put together that $\overline{Z}_t$ tends to a random final position a.s. with the description of the system 'as viewed from $\overline{Z}_t$?'
We certainly need:

Lemma

$Y = (Y_t; t \geq 0)$ is independent of the tail σ-algebra of $\overline{Z}$.

Asymptotic behavior for attraction, $\gamma > 0$

Theorem (E. 2010)

Let $P^x(\cdot) := P(\cdot \mid N = x)$. *As* $n \to \infty$,

$$2^{-n} Z_n(\mathrm{d}y) \overset{weak}{\Longrightarrow} \left(\frac{\gamma}{\pi}\right)^{d/2} \exp\left(-\gamma |y - x|^2\right) \mathrm{d}y, \; P^x - \text{a.s.}$$

for almost all $x \in \mathbb{R}^d$.

Asymptotic behavior for attraction, $\gamma > 0$

Theorem (E. 2010)

Let $P^x(\cdot) := P(\cdot \mid N = x)$. As $n \to \infty$,

$$2^{-n} Z_n(\mathrm{d}y) \stackrel{weak}{\Longrightarrow} \left(\frac{\gamma}{\pi}\right)^{d/2} \exp\left(-\gamma |y - x|^2\right) \mathrm{d}y, \ P^x - \text{a.s.}$$

for almost all $x \in \mathbb{R}^d$. Consequently,

$$2^{-n} E Z_n(\mathrm{d}y) \stackrel{weak}{\Longrightarrow} f^{\gamma}(y)\mathrm{d}y,$$

where

$$f^{\gamma}(\cdot) = \left(\pi(4 + \gamma^{-1})\right)^{-d/2} \exp\left[\frac{-|\cdot|^2}{4 + \gamma^{-1}}\right].$$

Asymptotic behavior for attraction, $\gamma > 0$

Theorem (E. 2010)

Let $P^x(\cdot) := P(\cdot \mid N = x)$*. As* $n \to \infty$*,*

$$2^{-n} Z_n(\mathrm{d}y) \overset{weak}{\Longrightarrow} \left(\frac{\gamma}{\pi}\right)^{d/2} \exp\left(-\gamma |y-x|^2\right) \mathrm{d}y, \ P^x - \text{a.s.}$$

for almost all $x \in \mathbb{R}^d$*. Consequently,*

$$2^{-n} E Z_n(\mathrm{d}y) \overset{weak}{\Longrightarrow} f^{\gamma}(y)\mathrm{d}y,$$

where

$$f^{\gamma}(\cdot) = \left(\pi(4+\gamma^{-1})\right)^{-d/2} \exp\left[\frac{-|\cdot|^2}{4+\gamma^{-1}}\right].$$

Remark

Variance corresponding to $f^{\gamma}(\cdot):\ \Sigma = \left(2 + \frac{1}{2\gamma}\right) \mathbf{I}_d$.

Stronger attraction $\to$ smaller variance.

Asymptotic behavior for repulsion, $\gamma < 0$

Conjecture

- *If* $|\gamma| \geq \frac{\log 2}{d}$, *then* Z *suffers local extinction:*

$$Z_n(\mathrm{d}y) \overset{vague}{\Longrightarrow} \mathbf{0}, \quad P - \text{a.s.}$$

- *If* $|\gamma| < \frac{\log 2}{d}$, *then*

$$2^{-n} e^{d|\gamma|n} Z_n(\mathrm{d}y) \overset{vague}{\Longrightarrow} \mathrm{d}y, \quad P - \text{a.s.}$$

COM for SBM

Theorem (E. 2010)

Let $\alpha, \beta > 0$ *and let*

$$\overline{X} := \frac{\langle \mathrm{id}, X \rangle}{\|X\|}$$

denote the C.O.M. for the $(\frac{1}{2}\Delta, \beta, \alpha; \mathbb{R}^d)$*-superdiffusion* X.

COM for SBM

Theorem (E. 2010)

Let $\alpha, \beta > 0$ *and let*

$$\overline{X} := \frac{\langle \mathrm{id}, X \rangle}{\|X\|}$$

denote the C.O.M. for the $(\frac{1}{2}\Delta, \beta, \alpha; \mathbb{R}^d)$*-superdiffusion* X*. Then, on the survival set,* $t \to \overline{X}_t$ *is continuous and converges* P_{δ_x}*-almost surely as* $t \to \infty$*.*

COM for SBM

Theorem (E. 2010)
Let $\alpha, \beta > 0$ and let

$$\overline{X} := \frac{\langle \mathrm{id}, X \rangle}{\|X\|}$$

denote the C.O.M. for the $(\frac{1}{2}\Delta, \beta, \alpha; \mathbb{R}^d)$-superdiffusion X. Then, on the survival set, $t \to \overline{X}_t$ is continuous and converges P_{δ_x}-almost surely as $t \to \infty$.

CHALLENGE: Generalize the interactive model and the result to SBM!

COM for SBM

Theorem (E. 2010)
Let $\alpha, \beta > 0$ *and let*

$$\overline{X} := \frac{\langle \mathrm{id}, X \rangle}{\|X\|}$$

denote the C.O.M. for the $(\frac{1}{2}\Delta, \beta, \alpha; \mathbb{R}^d)$*-superdiffusion* X*. Then, on the survival set,* $t \to \overline{X}_t$ *is continuous and converges* P_{δ_x}*-almost surely as* $t \to \infty$*.*

CHALLENGE: Generalize the interactive model and the result to SBM!

RESULTS: Very recent, interesting paper by Hardeep Gill.

Gill's work

Gill constructed a superprocess with attraction to its C.O.M.

Gill's work

Gill constructed a superprocess with attraction to its C.O.M.

Using Perkins's historical stochastic calculus, constructs a supercritical interacting measure-valued process with representative particles that are attracted to or repulsed from its C.O.M.

Gill's work

Gill constructed a superprocess with attraction to its C.O.M.

Using Perkins's historical stochastic calculus, constructs a supercritical interacting measure-valued process with representative particles that are attracted to or repulsed from its C.O.M.

Coupling between the ordinary super O-U process Z and the interacting process Z', constructed on the same probability space:

$$Z'_t = Z_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,$$

where $\overline{Z}_t$: C.O.M. of Z_t.

Gill's work

Gill constructed a superprocess with attraction to its C.O.M.

Using Perkins's historical stochastic calculus, constructs a supercritical interacting measure-valued process with representative particles that are attracted to or repulsed from its C.O.M.

Coupling between the ordinary super O-U process Z and the interacting process Z', constructed on the same probability space:

$$Z'_t = Z_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,$$

where $\overline{Z}_t$: C.O.M. of Z_t. In particular,

$$\boxed{\overline{Z}'_t = \overline{Z}_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,}$$

where $\overline{Z}'$ denotes the C.O.M. of Z'.

Gill's work

Gill constructed a superprocess with attraction to its C.O.M.

Using Perkins's historical stochastic calculus, constructs a supercritical interacting measure-valued process with representative particles that are attracted to or repulsed from its C.O.M.

Coupling between the ordinary super O-U process Z and the interacting process Z', constructed on the same probability space:

$$Z'_t = Z_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,$$

where $\overline{Z}_t$: C.O.M. of Z_t. In particular,

$$\boxed{\overline{Z}'_t = \overline{Z}_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,}$$

where $\overline{Z}'$ denotes the C.O.M. of Z'. Here γ = parameter of the underlying O-U process in Z = the parameter of attraction/repulsion for Z'.

Gill's work

Gill constructed a superprocess with attraction to its C.O.M.

Using Perkins's historical stochastic calculus, constructs a supercritical interacting measure-valued process with representative particles that are attracted to or repulsed from its C.O.M.

Coupling between the ordinary super O-U process Z and the interacting process Z', constructed on the same probability space:

$$Z'_t = Z_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,$$

where $\overline{Z}_t$: C.O.M. of Z_t. In particular,

$$\boxed{\overline{Z}'_t = \overline{Z}_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,}$$

where $\overline{Z}'$ denotes the C.O.M. of Z'. Here γ = parameter of the underlying O-U process in Z = the parameter of attraction/repulsion for Z'. For $\gamma < 0$ (repulsive case): 'outward' O-U.

(a) In the **attractive case**, Gill proves the equivalent of our theorem: on survival, the mass normalized process converges a.s. to the stationary distribution of the O-U process centered at the limiting value of its C.O.M.

(a) In the **attractive case**, Gill proves the equivalent of our theorem: on survival, the mass normalized process converges a.s. to the stationary distribution of the O-U process centered at the limiting value of its C.O.M.

(b) In the **repulsive case**, the equivalent of our conjecture is only partially demonstrated:

(a) In the **attractive case**, Gill proves the equivalent of our theorem: on survival, the mass normalized process converges a.s. to the stationary distribution of the O-U process centered at the limiting value of its C.O.M.

(b) In the **repulsive case**, the equivalent of our conjecture is only partially demonstrated:

(a) In the **attractive case**, Gill proves the equivalent of our theorem: on survival, the mass normalized process converges a.s. to the stationary distribution of the O-U process centered at the limiting value of its C.O.M.

(b) In the **repulsive case**, the equivalent of our conjecture is only partially demonstrated:

- convergence in probability is shown, provided the repulsion is not too strong compared to the mass creation, by appealing to a result of E. and Winter;

(a) In the **attractive case**, Gill proves the equivalent of our theorem: on survival, the mass normalized process converges a.s. to the stationary distribution of the O-U process centered at the limiting value of its C.O.M.

(b) In the **repulsive case**, the equivalent of our conjecture is only partially demonstrated:

- convergence in probability is shown, provided the repulsion is not too strong compared to the mass creation, by appealing to a result of E. and Winter;
- otherwise, local extinction is shown, however, only under the additional assumption that $|\gamma|$ is *also upper bounded* by a certain second constant.

Reason for the counterintuitive upper bound: otherwise it is not clear if $\overline{Z}'_t$ converges.

Reason for the counterintuitive upper bound: otherwise it is not clear if $\overline{Z}'_t$ converges.
Because of coupling: reduces to a problem about ordinary (non-interacting) super O-U processes, but it is apparently still a non-trivial question.

Reason for the counterintuitive upper bound: otherwise it is not clear if $\overline{Z}'_t$ converges.
Because of coupling: reduces to a problem about ordinary (non-interacting) super O-U processes, but it is apparently still a non-trivial question.

On extinction, a version of Tribe's result is proven: as $t \uparrow \xi_{ext}$, the normalized process in both the attractive and repulsive cases converges to the Dirac measure at a random point a.s.

The Balázs-Rácz-Tóth model (2011)

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

The Balázs-Rácz-Tóth model (2011)

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but

The Balázs-Rácz-Tóth model (2011)

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

The Balázs-Rácz-Tóth model (2011)

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

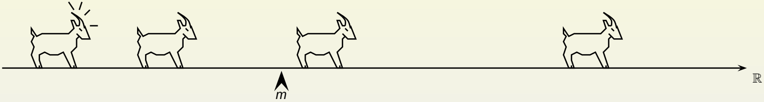
- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

Particles being far ahead slow down, while the laggards catch up.

Goats

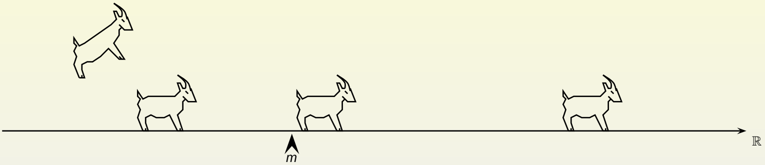
(Animation: courtesy of M. Balázs.)

Goats



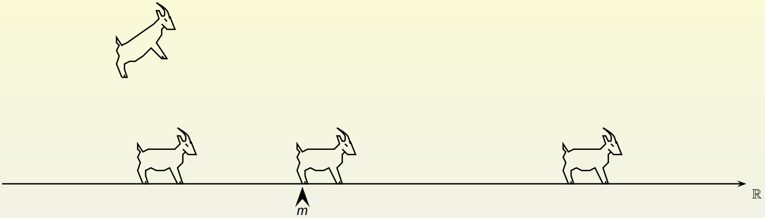
(Animation: courtesy of M. Balázs.)

Goats



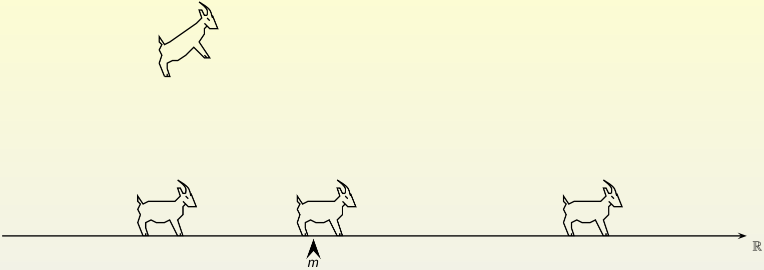
(Animation: courtesy of M. Balázs.)

Goats



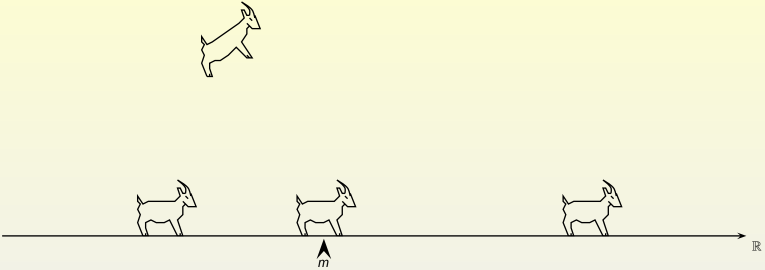
(Animation: courtesy of M. Balázs.)

Goats



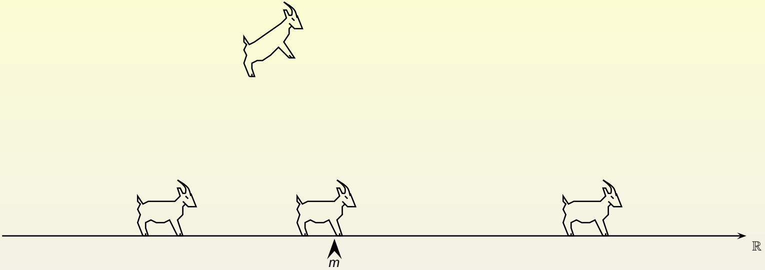
(Animation: courtesy of M. Balázs.)

Goats



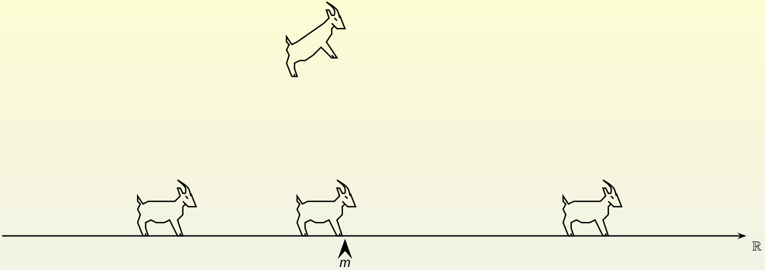
(Animation: courtesy of M. Balázs.)

Goats



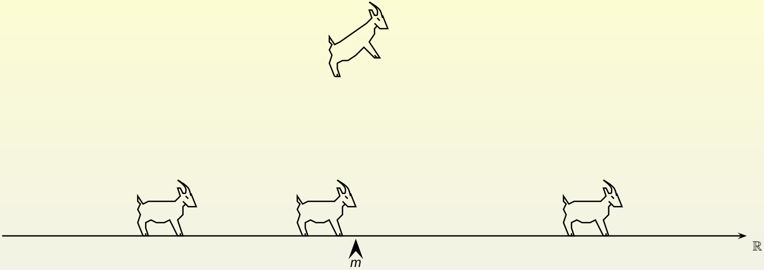
(Animation: courtesy of M. Balázs.)

Goats



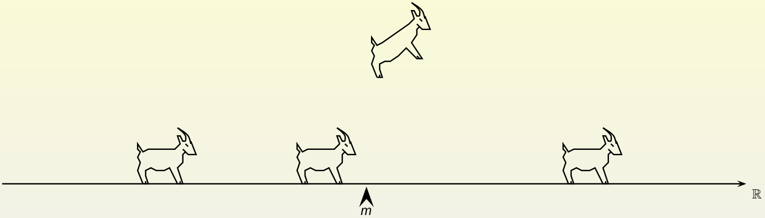
(Animation: courtesy of M. Balázs.)

Goats



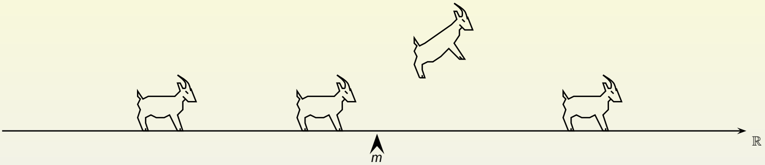
(Animation: courtesy of M. Balázs.)

Goats



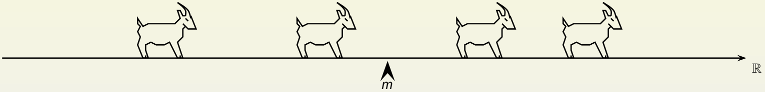
(Animation: courtesy of M. Balázs.)

Goats



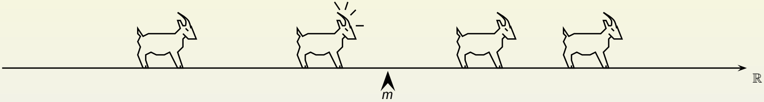
(Animation: courtesy of M. Balázs.)

Goats



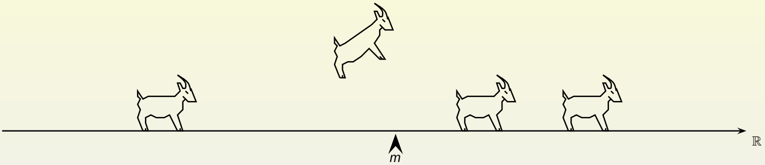
(Animation: courtesy of M. Balázs.)

Goats



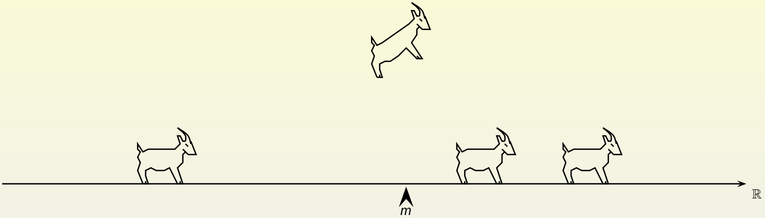
(Animation: courtesy of M. Balázs.)

Goats



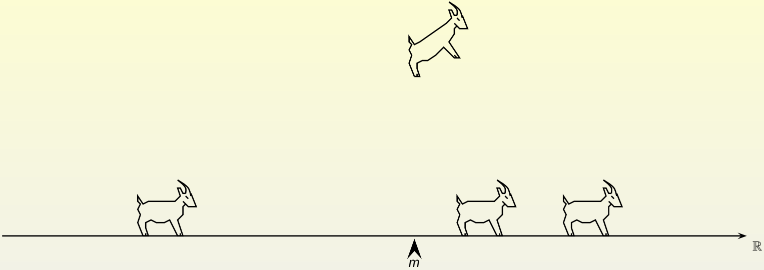
(Animation: courtesy of M. Balázs.)

Goats



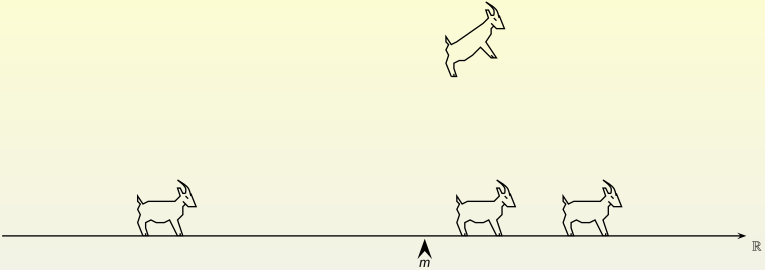
(Animation: courtesy of M. Balázs.)

Goats



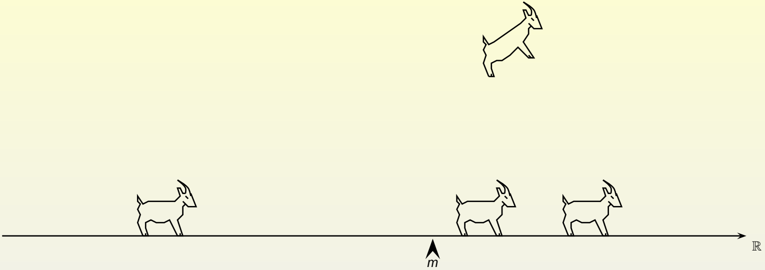
(Animation: courtesy of M. Balázs.)

Goats



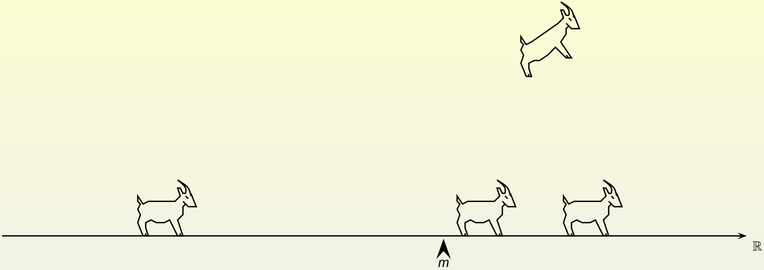
(Animation: courtesy of M. Balázs.)

Goats



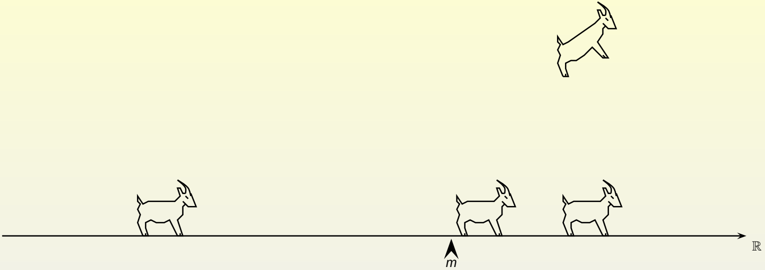
(Animation: courtesy of M. Balázs.)

Goats



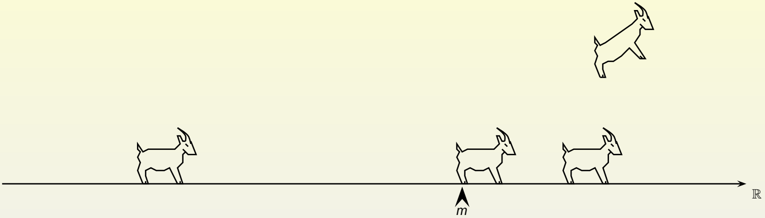
(Animation: courtesy of M. Balázs.)

Goats



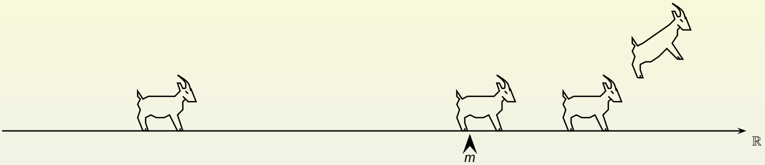
(Animation: courtesy of M. Balázs.)

Goats



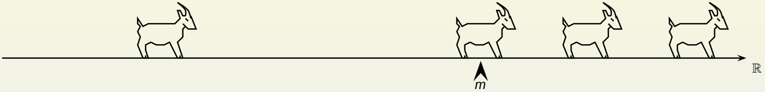
(Animation: courtesy of M. Balázs.)

Goats



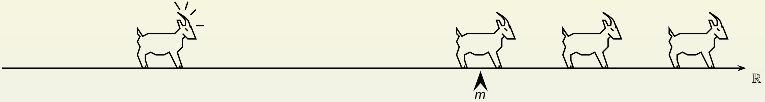
(Animation: courtesy of M. Balázs.)

Goats



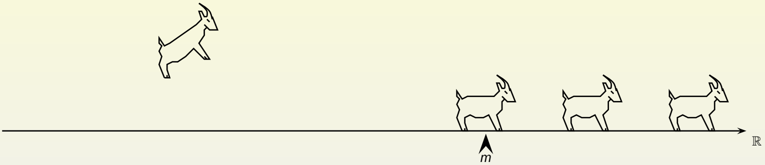
(Animation: courtesy of M. Balázs.)

Goats



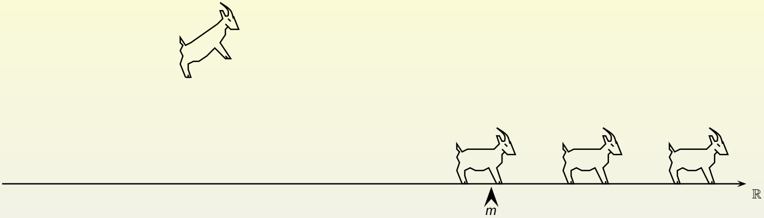
(Animation: courtesy of M. Balázs.)

Goats



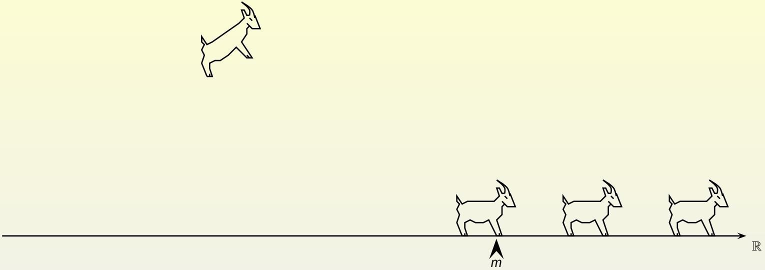
(Animation: courtesy of M. Balázs.)

Goats



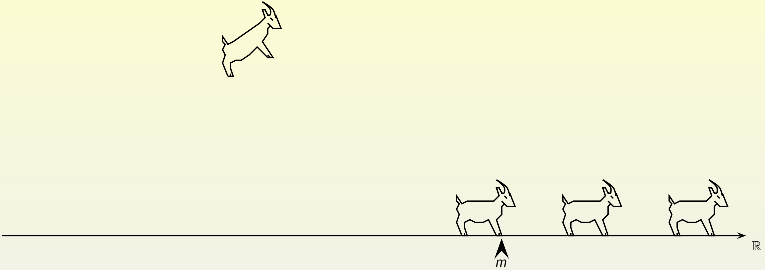
(Animation: courtesy of M. Balázs.)

Goats



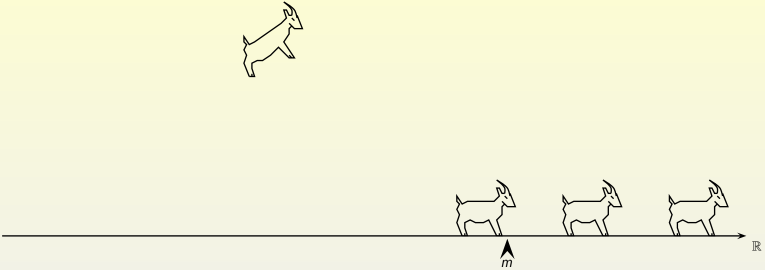
(Animation: courtesy of M. Balázs.)

Goats



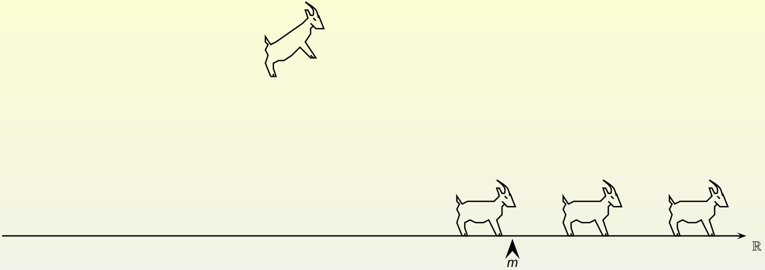
(Animation: courtesy of M. Balázs.)

Goats



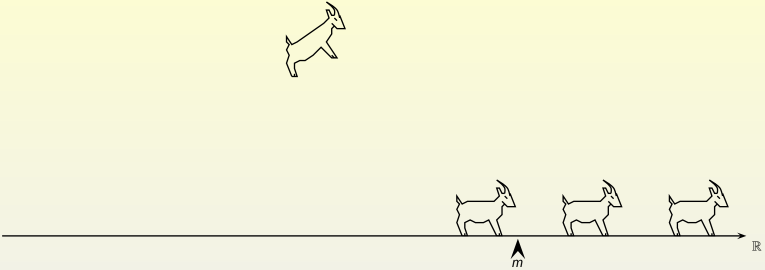
(Animation: courtesy of M. Balázs.)

Goats



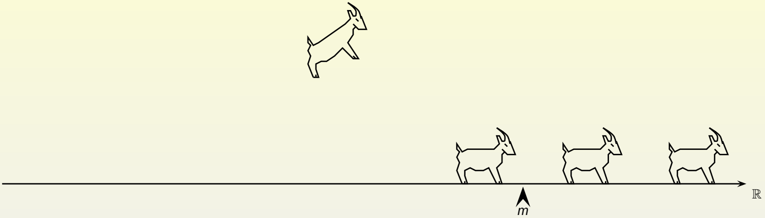
(Animation: courtesy of M. Balázs.)

Goats



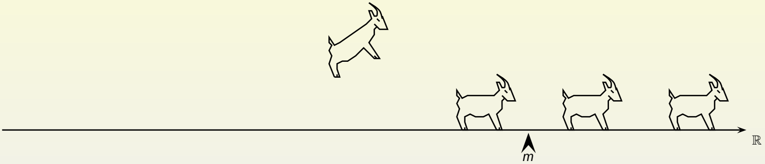
(Animation: courtesy of M. Balázs.)

Goats



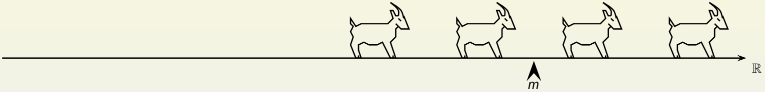
(Animation: courtesy of M. Balázs.)

Goats



(Animation: courtesy of M. Balázs.)

Goats



(Animation: courtesy of M. Balázs.)

The B-R-T model

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

Particles being far ahead slow down, while the laggards catch up.

The B-R-T model

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

Particles being far ahead slow down, while the laggards catch up.

RESULT: There is a limiting probability measure-valued process around the C.O.M., as number of particles tends to infinity and individual particle mass is taken $1/n$.

The B-R-T model

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

Particles being far ahead slow down, while the laggards catch up.

RESULT: There is a limiting probability measure-valued process around the C.O.M., as number of particles tends to infinity and individual particle mass is taken $1/n$.
Limiting measure valued process is deterministic.

The B-R-T model

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

Particles being far ahead slow down, while the laggards catch up.

RESULT: There is a limiting probability measure-valued process around the C.O.M., as number of particles tends to infinity and individual particle mass is taken $1/n$.
Limiting measure valued process is deterministic.

CHALLENGE: Introduce branching and get interacting superprocess in the limit.

Thank you!

Why $\log 2/d$?

Why $\log 2/d$?

For a branching diffusion on $\mathbb{R}^d$ with motion generator L, smooth nonzero spatially dependent exponential branching rate $\beta(\cdot) \geq 0$ and dyadic branching: either local extinction or local exponential growth according to whether $\lambda_c \leq 0$ or $\lambda_c > 0$.

($\lambda_c = \lambda_c(L + \beta)$: *generalized principal eigenvalue* of $L + \beta$ on $\mathbb{R}^d$.)

Why $\log 2/d$?

For a branching diffusion on $\mathbb{R}^d$ with motion generator L, smooth nonzero spatially dependent exponential branching rate $\beta(\cdot) \geq 0$ and dyadic branching: either local extinction or local exponential growth according to whether $\lambda_c \leq 0$ or $\lambda_c > 0$.

($\lambda_c = \lambda_c(L + \beta)$: *generalized principal eigenvalue* of $L + \beta$ on $\mathbb{R}^d$.)

When $\beta \equiv B > 0$, the criterion for local exponential growth becomes $B > |\lambda_c(L)|$.

Why $\log 2/d$?

For a branching diffusion on $\mathbb{R}^d$ with motion generator L, smooth nonzero spatially dependent exponential branching rate $\beta(\cdot) \geq 0$ and dyadic branching: either local extinction or local exponential growth according to whether $\lambda_c \leq 0$ or $\lambda_c > 0$.

($\lambda_c = \lambda_c(L + \beta)$: *generalized principal eigenvalue* of $L + \beta$ on $\mathbb{R}^d$.)

When $\beta \equiv B > 0$, the criterion for local exponential growth becomes $B > |\lambda_c(L)|$.

Note: $\lambda_c(L)$ is the "exponential escape rate from compacts" for the diffusion corresponding to L. A large enough mass creation can compensate that particles drift away from a given bounded set.
(If $L \sim$ recurrent diffusion then $\lambda_c(L) = 0$.)

Why $\log 2/d$?

For a branching diffusion on $\mathbb{R}^d$ with motion generator L, smooth nonzero spatially dependent exponential branching rate $\beta(\cdot) \geq 0$ and dyadic branching: either local extinction or local exponential growth according to whether $\lambda_c \leq 0$ or $\lambda_c > 0$.

($\lambda_c = \lambda_c(L + \beta)$: *generalized principal eigenvalue* of $L + \beta$ on $\mathbb{R}^d$.)

When $\beta \equiv B > 0$, the criterion for local exponential growth becomes $B > |\lambda_c(L)|$.

Note: $\lambda_c(L)$ is the "exponential escape rate from compacts" for the diffusion corresponding to L. A large enough mass creation can compensate that particles drift away from a given bounded set. (If $L \sim$ recurrent diffusion then $\lambda_c(L) = 0$.)

In our case: $\lambda_c = d\gamma$ for the outward O-U, and for unit time branching, the role of B is played by $\log 2$. The condition for local exponential growth: $\log 2 > d|\gamma|$.