Particle models with interaction through the center of mass

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In this talk,

C.O.M.: Center of mass

Model

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($n_t := 2^{\lfloor t \rfloor}$, where $\lfloor t \rfloor$ is the integer part of t.)
If $\gamma > 0$: attraction; if $\gamma < 0$: repulsion.

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$$\frac{1}{n_t} \sum_{1 \le j \le n_t} g\left(|Z_t^j - \cdot|\right) \frac{Z_t^j - \cdot}{|Z_t^j - \cdot|},$$

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- **H. Gill** (U. British Columbia): super-Brownian motion with self-interaction
- **M. Balázs**, **B. Tóth** and **M. Rácz** (Budapest Technical U.): A particle system interacting through the C.O.M.

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Lemma (C.O.M. stabilizes)

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Remark: In fact, $\overline{Z}$ *is a Markov process* w.r. to canonical filtration for Z.

Viewing the system from C.O.M.

Assume that $t \in [m, m+1)$. When viewed from $\overline{Z}$, the relocation of a particle is governed by

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The RHS is a Brownian motion with mean zero and variance $(1 - 2^{-m})\tau \mathbf{I}_d := \sigma_m^2 \tau \mathbf{I}_d$.

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We certainly need:

Lemma

$Y = (Y_t; t \geq 0)$ is independent of the tail σ-algebra of $\overline{Z}$.

Asymptotic behavior for attraction, $\gamma > 0$

Theorem (E. 2010)

Let $P^x(\cdot) := P(\cdot \mid N = x)$*. As* $n \to \infty$*,*

$$2^{-n} Z_n(\mathrm{d}y) \overset{weak}{\Longrightarrow} \left(\frac{\gamma}{\pi}\right)^{d/2} \exp\left(-\gamma |y-x|^2\right) \mathrm{d}y, \; P^x - \text{a.s.}$$

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Remark

Variance corresponding to $f^\gamma(\cdot)$: $\Sigma = \left(2 + \frac{1}{2\gamma}\right) \mathbf{I}_d$.

Stronger attraction $\to$ smaller variance.

Asymptotic behavior for repulsion, $\gamma < 0$

Conjecture

- *If* $|\gamma| \geq \frac{\log 2}{d}$*, then* Z *suffers local extinction:*

$$Z_n(\mathrm{d}y) \overset{vague}{\Longrightarrow} \mathbf{0}, \quad P - \text{a.s.}$$

- *If* $|\gamma| < \frac{\log 2}{d}$*, then*

$$2^{-n} e^{d|\gamma|n} Z_n(\mathrm{d}y) \overset{vague}{\Longrightarrow} \mathrm{d}y, \quad P - \text{a.s.}$$

COM for SBM

Theorem (E. 2010)
Let $\alpha, \beta > 0$ *and let*

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denote the C.O.M. for the $(\frac{1}{2}\Delta, \beta, \alpha; \mathbb{R}^d)$*-superdiffusion* X.

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CHALLENGE: Generalize the interactive model and the result to SBM!

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RESULTS: Very recent, interesting paper by Hardeep Gill.

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Coupling between the ordinary super O-U process Z and the interacting process Z', constructed on the same probability space:

$$Z'_t = Z_t + \gamma \int_0^t \overline{Z}_s \, \mathrm{d}s,$$

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where $\overline{Z}'$ denotes the C.O.M. of Z'. Here γ = parameter of the underlying O-U process in Z = the parameter of attraction/repulsion for Z'. For $\gamma < 0$ (repulsive case): 'outward' O-U.

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(b) In the **repulsive case**, the equivalent of our conjecture is only partially demonstrated:

- convergence in probability is shown, provided the repulsion is not too strong compared to the mass creation, by appealing to a result of E. and Winter;
- otherwise, local extinction is shown, however, only under the additional assumption that $|\gamma|$ is *also upper bounded* by a certain second constant.

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On extinction, a version of Tribe's result is proven: as $t \uparrow \xi_{ext}$, the normalized process in both the attractive and repulsive cases converges to the Dirac measure at a random point a.s.

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One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

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Particles being far ahead slow down, while the laggards catch up.

Goats

(Animation: courtesy of M. Balázs.)

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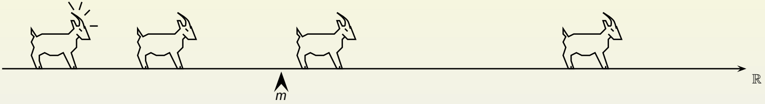

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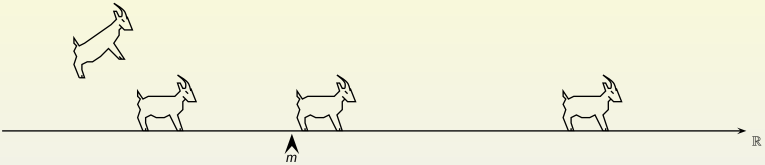

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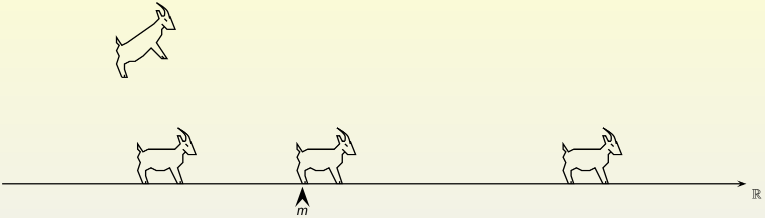

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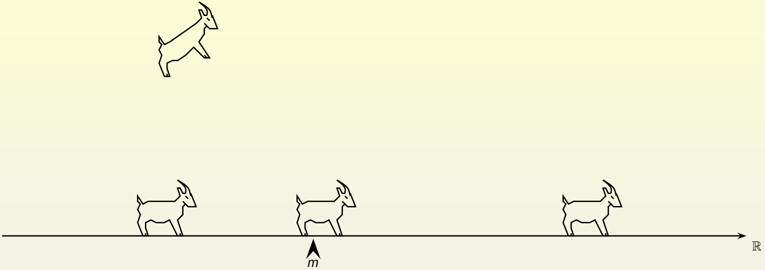

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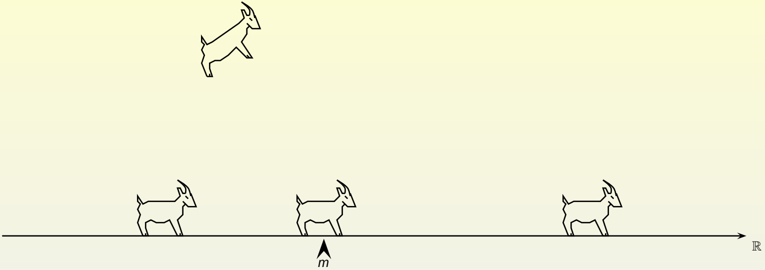
Goats



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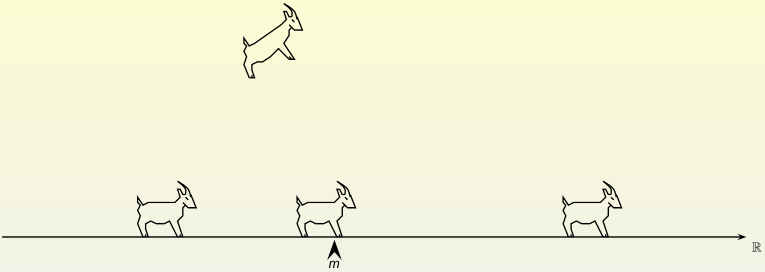

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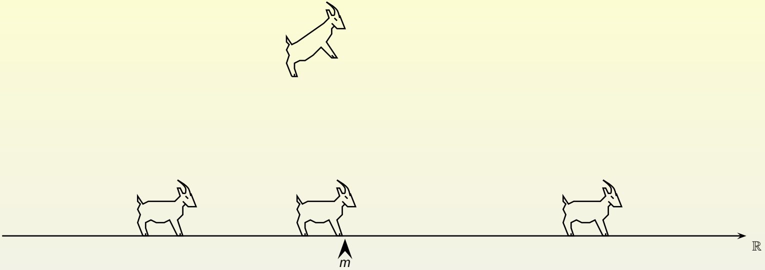

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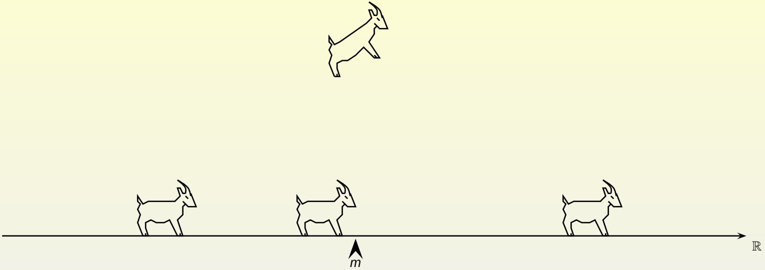

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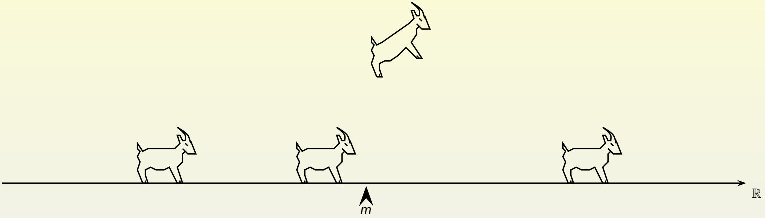

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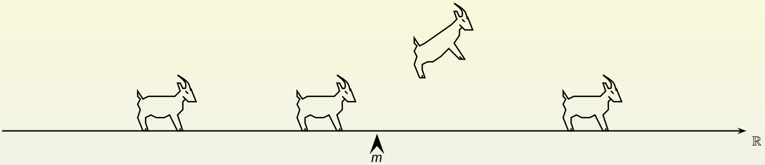

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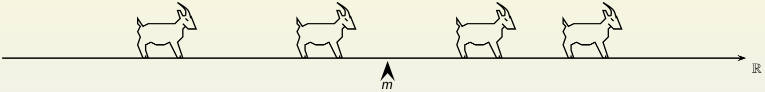

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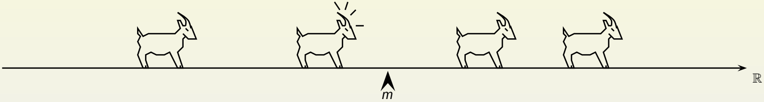

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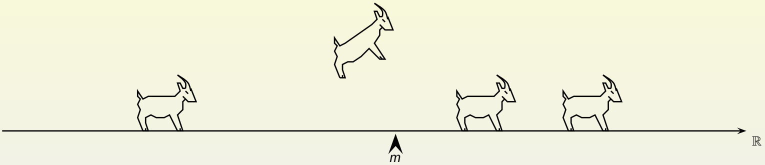

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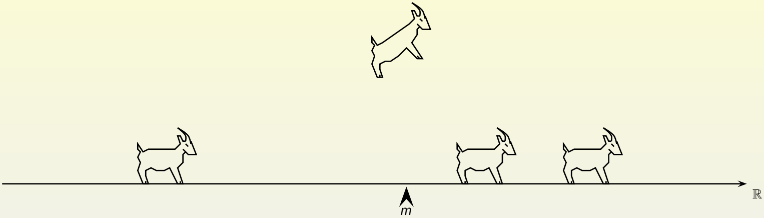

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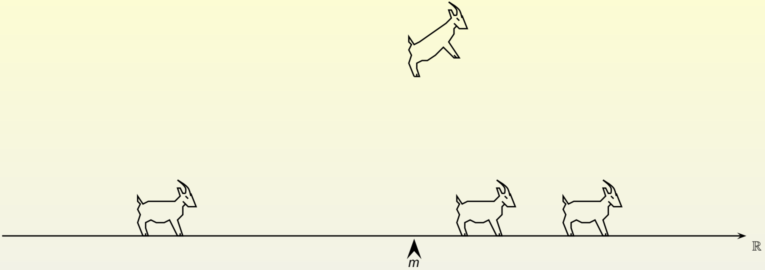

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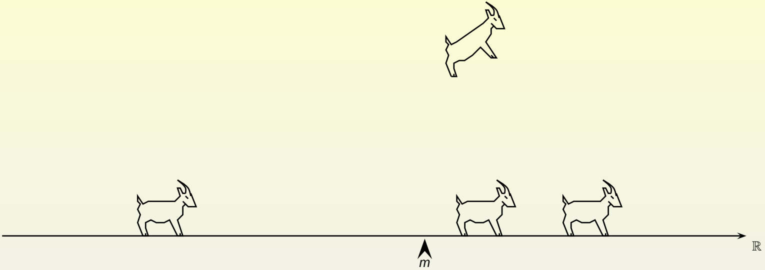

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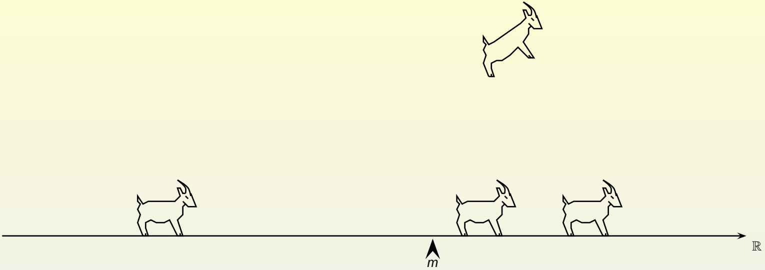

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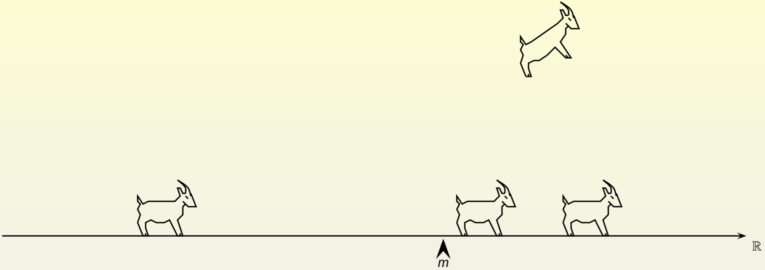

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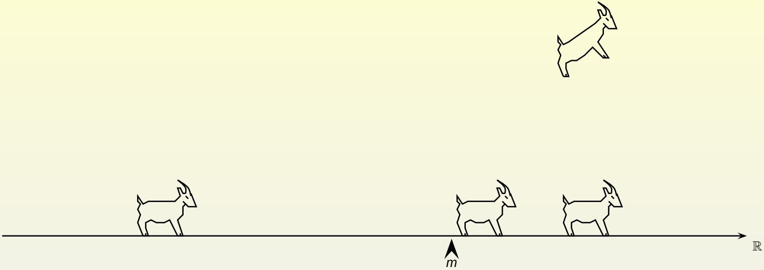

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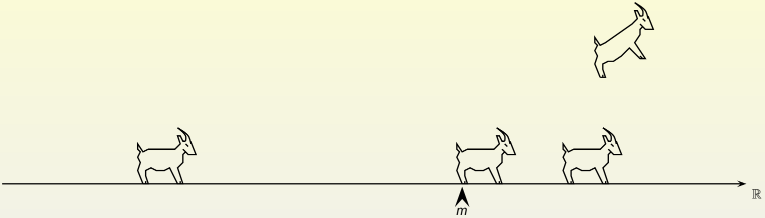

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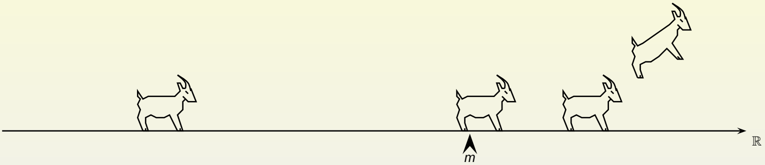

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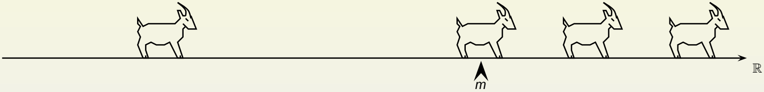

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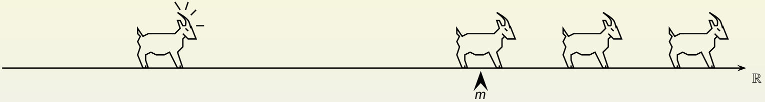

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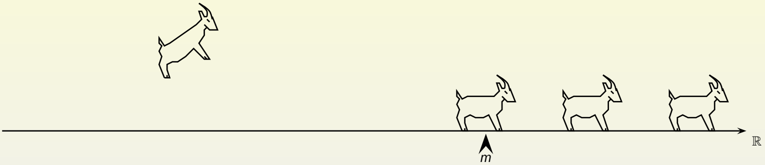

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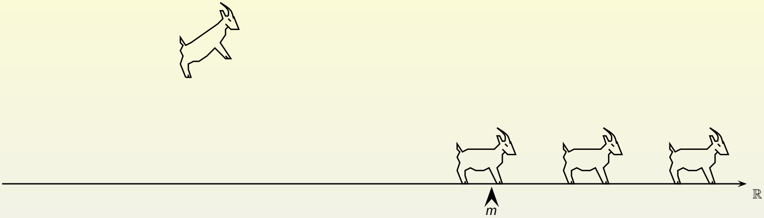

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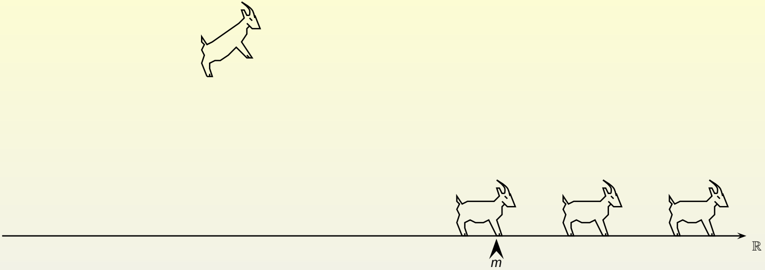

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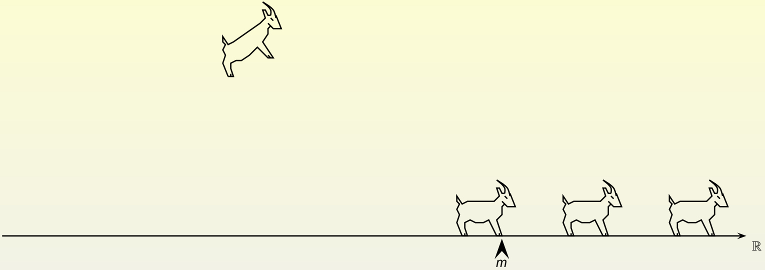

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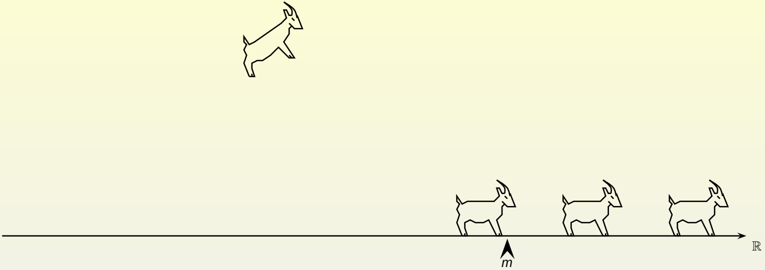

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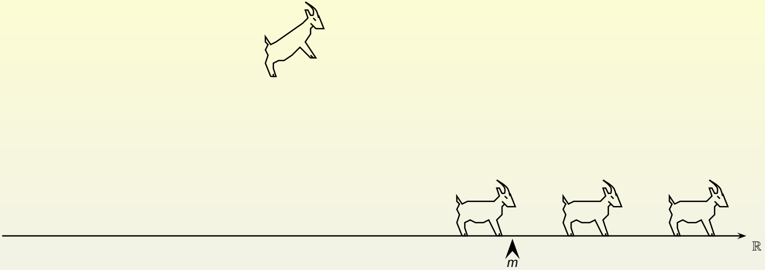

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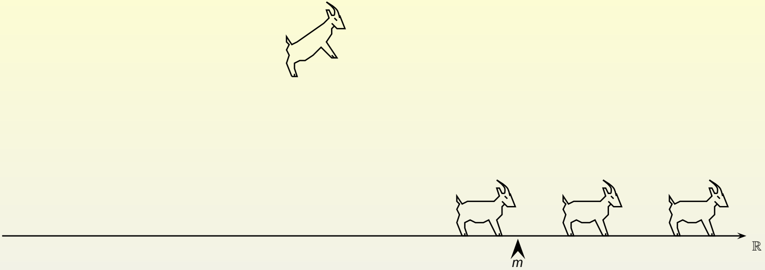

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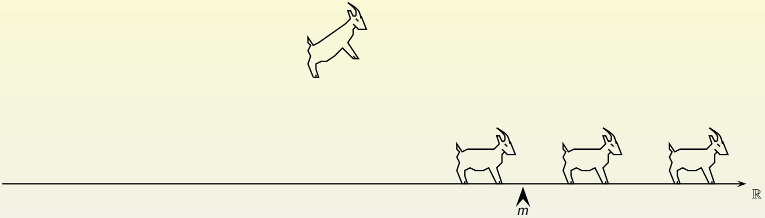

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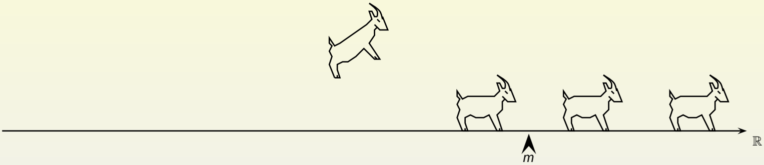

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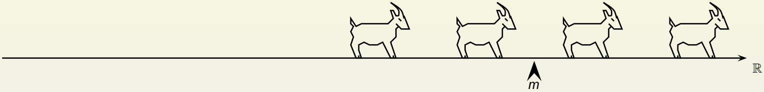

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The B-R-T model

One dimensional particle system with interaction via C.O.M. ("competing stocks model", or "goats").

There is a kind of attraction towards the C.O.M. in the following sense:

- each particle jumps to the right according to some common distribution F, but
- the rate at which the jump occurs is a monotone decreasing function of the signed distance between the particle and the mass center.

Particles being far ahead slow down, while the laggards catch up.

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CHALLENGE: Introduce branching and get interacting superprocess in the limit.

Thank you!

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For a branching diffusion on $\mathbb{R}^d$ with motion generator L, smooth nonzero spatially dependent exponential branching rate $\beta(\cdot) \geq 0$ and dyadic branching: either local extinction or local exponential growth according to whether $\lambda_c \leq 0$ or $\lambda_c > 0$.

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In our case: $\lambda_c = d\gamma$ for the outward O-U, and for unit time branching, the role of B is played by $\log 2$. The condition for local exponential growth: $\log 2 > d|\gamma|$.