Problem 1 Let $W(t)$ be standard Brownian motion. Compute the following probabilities. I want an exact answer, not an approximation. So you can use simulations to get an approximation of the probability and thus verify your answers, if you wish. But you cannot give them as THE answer. I want to see the calculus. You can of course round off the answer.

a) $P\{W(2) > 2\}$,
b) $P\{W(2) > W(1)\}$,
c) $P\{W(2) > W(1) > W(3)\}$,
d) $E[(W(4) - W(2))W(2)]$,
e) $E[W(2)^2]$,
f) $E[W(2)W(4)]$.

Problem 2 Assume $B(t)$ is a Brownian motion with diffusion coefficient $\sigma^2$. Let $a > 0$.

a) Show that $X(t) = B(at)$ is a Brownian motion with diffusion coefficient $a\sigma^2$.
b) Show that $Y(t) = aB(t)$ is a Brownian motion with diffusion coefficient $a^2\sigma^2$.
c) Show that $Z(t) = a^{-1/2}B(at)$ is a Brownian motion with diffusion coefficient $\sigma^2$.

Problem 3 Let $W(t)$ and $B(t)$ be independent standard Brownian motions.

a) Show that $X(t) = W(t) - B(t)$ is a Brownian motion. What is its diffusion coefficient?
b) True or False: With probability one, $W(t) = B(t)$ for infinitely many values of $t$. 