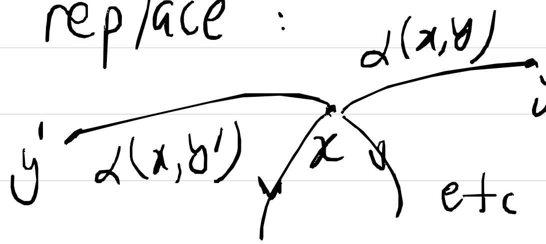
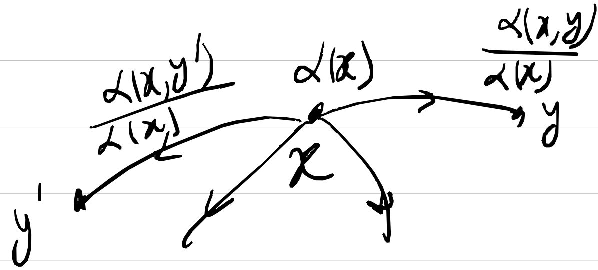
Lec 28

So we can replace:



by:



$\alpha(x) = \sum_{z \in S} \alpha(x,z)$

Clocks are now on sites instead of arrows. When ~~the~~ the site's clock rings, the chain moves.

The probabilities on the arrows tell us where to move to.

Can we use a single clock?

Yes, if $\sup_{x \in S} \alpha(x) < \infty$

↑ always the case is S is finite
not so, if not.

Suppose T_j are iid $\exp(\alpha)$

$0 < \alpha \leq a$

$N \sim \text{Geo}\left(\frac{\alpha}{a}\right)$

Let $S = T_1 + \cdots + T_N$

(each time the α-clock rings, flip a $p = \frac{\alpha}{a}$ -coin to decide if "wake up" or not. S = time when wake up)

Prob. of H

Think of tossing a q-coin repeatedly.

Suppose $p \leq q$

When H occurs, toss a $\frac{p}{q}$ coin to decide if we keep the H or flip it back to a T.

In total, this is like a p-coin tossing and waiting for an H.

(P(success in combined process)
$= P(\text{H in } q\text{-coin}) \cdot P(\text{H in } \tfrac{p}{q} \text{ coin})$)

$= q \cdot \frac{p}{q} = p$

to keep the q-coin's H

Pf.

$$E[e^{tS}] = E[e^{t(T_1+\cdots+T_N)}]$$

$$= \sum_{n=1}^{\infty} E[e^{t(T_1+\cdots+T_n)}]\, P(N=n)$$

$$= \sum_{n=1}^{\infty} E[e^{tT}]^n\, P(N=n)$$

$$= \sum_{n=1}^{\infty} \left(\frac{a}{a-t}\right)^n \cdot (1-p)^{n-1} p$$

$$= \frac{p}{1-p} \sum_{n=1}^{\infty} \left(\frac{a(1-p)}{a-t}\right)^n$$

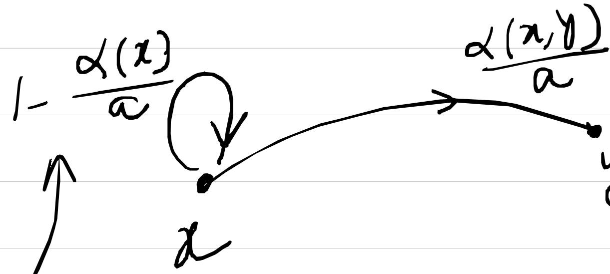
$$= \frac{p}{1-p} \frac{\frac{a(1-p)}{a-t}}{1-\frac{a(1-p)}{a-t}} = \frac{p}{\cancel{1-p}} \frac{a\cancel{(1-p)}}{\underbrace{a-t-a(1-p)}_{ap-t}}$$

$ap=\alpha$

$$= \frac{ap}{ap-t} = mgf_{exp(\alpha)}(t) \quad \text{so } S \sim exp(\alpha).$$

So one can use a single clock w/ rate $a \geq \sup_{x \in S} \alpha(x)$

prob. of jumping to y when clock rings



prob. of "ignoring" the clock

Note: if x is s.t. $\alpha(x) = a$, then $1 - \frac{\alpha(x)}{a} = 0$. So jump happens when clock rings.

while if $\alpha(x) < a$: $1 - \frac{\alpha(x)}{a} > 0$

and jump happens at lower rate than a (indeed, rate $\alpha(x)$) b/c a-clock is ignored w/ prob. $1 - \frac{\alpha(x)}{a}$

[makes sense: site x has clock w/ rate $\alpha(x)$ but common clock is ringing at rate $a > \alpha(x)$, i.e. faster]