Lec 19

trans. vs. Rec. : α v.s. no α

pos. rec. vs. null rec. : π v.s. no π

if we think it is pos. rec. :
we look for π

if we think it is trans. :
we look for α

the hardest case

if no α and no π, then null rec.

α solves:
- $\alpha(z) = 1$ &
- $0 \le \alpha(x) \le 1 \quad \forall x$
- $\alpha(x) = \sum_{y \in S} P(x,y)\,\alpha(y) \quad \forall x \neq z$
- $\alpha(x) \to 0$ as x goes far

π solves:
- $0 \le \pi(x) \le 1 \quad \forall x$
- $\sum_{y \in S} \pi(y) = 1$
- $\pi(x) = \sum P(y,x)\,\pi(y)$

$\left(\underline{\Phi}_\infty = \underline{\Phi}_\infty P\right) \to$

SSRW $d = 1$ or 2 rec. but $P_{2n}(0,0) = \frac{c}{\sqrt{n}}$ or $\left(\frac{c}{\sqrt{n}}\right)^2$

if we had a π we would have $\pi(x) = \lim P(x,x)$
$= \lim P_n(0,0)$
$= 0$

so no inv. measure
i.e. null rec.

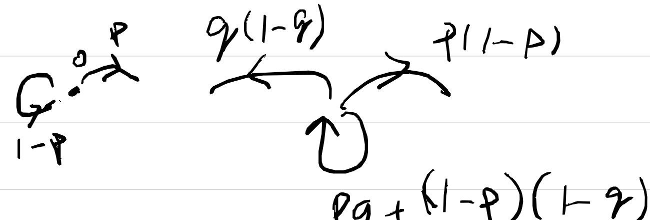
Also $\pi(x) = \frac{1}{2}\pi(x-1) + \frac{1}{2}\pi(x+1)$
linear difference w/ unique root $= 1$

so $\pi(x) = A + Bx$

$0 \le \pi(x) \le 1 \Rightarrow B = 0$ $\quad \pi(x) = A \ \forall x$
can't add up to 1!

so no π indeed.

<u>Queue:</u>

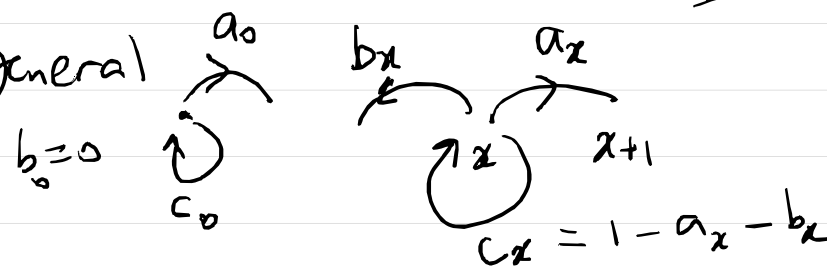


p: customer arrives
q: customer leaves

Guess: $p > q$ transient
$p < q$ pos. rec.
$p = q$ null rec.?

<u>birth & death process</u>

More general



For transience:

$\alpha(x) = a_x \alpha(x+1) + b_x \alpha(x-1) + c_x \alpha(x) \quad x \geq 1$

$\alpha(0) = 1 \quad (z = 0)$

$= a_x \alpha(x) + b_x \alpha(x) + c_x \alpha(x)$

<u>NOT</u> a difference equation.
Need some other way to solve.