Lec 17

$$P_{2n}(0,0) = \sum_{i+j+k=n} \frac{(2n)!}{(i!\,j!\,k!)^2}\cdot\frac{1}{6^{2n}}$$

$$= \sum_{i+j+k=n}\binom{2n}{n}\left(\frac{n!}{i!\,j!\,k!}\right)^2\cdot\frac{1}{6^{2n}}$$

$$= \binom{2n}{n}\frac{1}{2^{2n}}\sum_{i+j+k=n}\binom{n}{i,j,k}^2\cdot\frac{1}{3^{2n}}$$

n slots

w/ i red

j blue

k green

so 3^n

Consider $n=3m$

<u>Claim:</u> $\binom{3m}{i,j,k} \le \binom{3m}{m,m,m}$

<u>Pf</u> $$\frac{(3m)!}{(i-1)!\,(j+1)!\,k!} = \frac{(3m)!}{i!\,j!\,k!}\,\underbrace{\frac{i}{j+1}}_{<1} \quad \text{if } i\le j$$

so highest when $i=j=k=m$

Then $$P_{2n}(0,0) \le \binom{2n}{n}\cdot\frac{1}{2^{2n}}\cdot\frac{1}{3^n}\binom{3m}{m\,m\,m}\sum_{i+j+k=n}\binom{n}{i\,j\,k}\cdot\frac{1}{3^n}$$

$$\frac{(2n)!}{(n!)^2}\cdot\frac{1}{2^{2n}}\cdot\frac{1}{3^n}\cdot\frac{n!}{(m!)^3}$$

$$\frac{c\sqrt{2n}\,\cancel{(2n)^{2n}}\,\cancel{e^{-2n}}}{c\sqrt{n}\,\cancel{n^n}\,\cancel{e^{-n}}}\cdot\frac{1}{\cancel{2^{2n}}}\cdot\frac{1}{3^n}\cdot\frac{1}{(c\sqrt{m}\,m^m\,\cancel{e^{-m}})^3} \approx \frac{c'}{n^{3/2}}$$

$m=\frac{n}{3}$

So $\sum_{m} P_{6m}(0,0) < \infty$

what about $\sum P_{2(3m+1)}(0,0)$ and $\sum P_{2(3m+2)}(0,0)$?

0 $\quad 2(3m+1)$ steps

← total of $2(3(m+1))$ steps has probability

$$\le P_{2(3(m+1))}(0,0)$$

So $\sum_{m=0}^{\infty} P_{2(3m+1)}(0,0) \le \sum_{m=1}^{\infty} P_{6m}(0,0) < \infty$

same w/ $\sum_{m=0}^{\infty} P_{2(3m+2)}(0,0) \le \sum_{m=1}^{\infty} P_{6m}(0,0) < \infty$

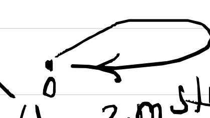
So $\sum_{n=0}^{\infty} P_{2n}(0,0) < \infty$ Transient

2d: Guess: $P_{2n}(0,0) \sim \left(\frac{c}{\sqrt{n}}\right)^2 = \frac{c'}{n}$ $\quad \Sigma = \infty$ so rec.

to prove it, enough to show

$\sum P_{2n}(0,0) \ge$ Something that Σ to ∞

e.g.

$\sum P_{2n}(0,0) \ge \sum P_{4n}(0,0) \ge \sum_{n=1}^{\infty} P\Big($  $\Big)$

2m steps

2m steps

↑ multiples of 2 ↑ multiples of 4 (fewer)

$P_{2n}(0,0)^2 \sim \frac{c}{n}$