$$\boxed{a(j) = \frac{1-\left(\frac{1-p}{p}\right)^j}{1-\left(\frac{1-p}{p}\right)^N}}$$ (note: always > 0)

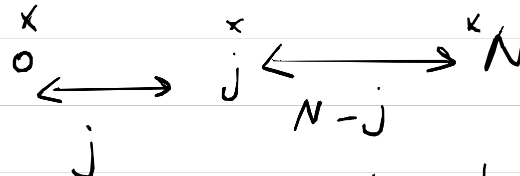
$p = \frac{1}{2}$: One solution $u = 1$

so $a(j) = \alpha + \beta j$

$0 = a(0) = \alpha$ so $\alpha = 0$

$1 = a(N) = \beta N$ so $\beta = \frac{1}{N}$

$$\boxed{a(j) = \frac{j}{N}}$$



Lec14

In this case, let T be # steps till walk reaches 0 or N

$G(j) = E[T \mid X_0 = j] = ?$

Could use Q^{-1} but N unknown.

$G(0) = 0 \qquad G(N) = 0$

$$G(j) = \frac{1}{2}\left(G(j+1)+1\right) + \frac{1}{2}\left(G(j-1)+1\right)$$

$$= \frac{1}{2}G(j+1) + \frac{1}{2}G(j-1) + 1$$

messes up linear differences

But: $\frac{1}{2}G(j)+\frac{1}{2}G(j)=\frac{1}{2}G(j+1)+\frac{1}{2}G(j-1)+1$

so

$$\frac{1}{2}\Big(\underbrace{G(j+1)-G(j)}_{b(j)}\Big)-\frac{1}{2}\Big(\underbrace{G(j)-G(j-1)}_{b(j-1)}\Big)=-1\,!$$

$$b(j)=b(j)-b(j-1)+b(j-1)-b(j-2)$$
$$+\cdots+b(1)-b(0)+b(0)$$

$$=-2-2-\cdots-2+b(0)$$
$$=-2j+b(0)$$

← call this c for now

$$G(j+1)-G(j)=-2j+c$$

$$G(j)=G(j)-G(j-1)+G(j-1)-G(j-2)$$
$$+\cdots+G(1)-G(0)+G(0)$$

$$=[-2(j-1)+c]+[-2(j-2)+c]+\cdots$$
$$+\cdots+[-2\cdot 0+c]+G(0)$$

← $=0$

$$=-2\underbrace{(0+1+\cdots+j-1)}_{\frac{j(j-1)}{2}}+cj$$

$G(N)=0 \Rightarrow$

$-N^2+N+cN=0$

$$=-j^2+j+cj$$

so $c=N-1$

$$\boxed{G(j)=-j^2+Nj=j(N-j)}$$

Infinitely countable state space

e.g. $1, 2, 3, 4, \dots$

or $0, 1, 2, 3, 4, \dots$

or $\dots, -2, -1, 0, 1, 2, 3, \dots$

or $\mathbb{Z}^2$ or $\mathbb{Z}^d$

Not $\mathbb{R}$ or $[0,1)$

MC defined similarly, but now

no transition matrix:

$$p(i,j) = P(X_{n+1} = j \mid X_n = i)$$

$$\sum_{j \in S} p(i,j) = 1 \quad \forall i \in S$$

like a $\infty \times \infty$ matrix :)

n-step transition: $P_n(i,j) = P(X_n = j \mid X_0 = i)$

No P^n matrix. Instead ($\Leftrightarrow P^{n+m} = P^n P^m$)

$$P_{n+m}(i,j) = \sum_{k \in S} P_n(i,k) P_m(k,j)$$ ← Chapman Kolmogorov