

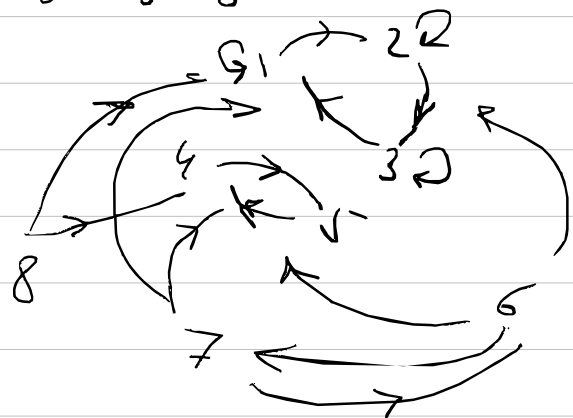
Lec 12

Example

	1	2	3	4	5	6	7	8
1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0
2	0	$\frac{1}{3}$	$\frac{2}{3}$	0	0	0	0	0
3	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	0	0
4	0	0	0	0	1	0	0	0
5	0	0	0	1	0	0	0	0
6	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0
7	0	0	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	0	$\frac{1}{8}$
8	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	0	0

Classes : $\{1, 2, 3\}$
 $\{4, 5\}$
 $\{6, 7\}$ $\{8\}$

	①	②
①	1	0
②	0	1
6	$\frac{1}{3}$	$\frac{1}{3}$
7	$\frac{1}{2}$	$\frac{1}{4}$
8	$\frac{1}{2}$	$\frac{1}{2}$



$$M = (I - Q) = \begin{array}{c|ccc} & 6 & 7 & 8 \\ \hline 1 & 0.4 & 0.3 & 0.3 \\ 2 & 0.1 & 0.4 & 0.5 \\ 3 & 0.1 & 0.4 & 0.5 \end{array}$$

8 leaves right away!

$$M_S = \begin{array}{c|cc} & ① & ② \\ \hline 6 & 0.5 & 0.5 \\ 7 & 0.6 & 0.4 \\ 8 & 0.5 & 0.5 \end{array}$$

obvious $\rightarrow$

avg. fraction of time at 3 starting at 6?

$$0.5435 \times \Phi_{\infty}(3)$$

↑
 $P(\text{get to class of 3} \mid X_0 = 6)$ fraction of time at 3 once MC is in class of 3

Φ_{∞} for 3:

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \end{matrix} \rightsquigarrow \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & -\frac{2}{3} & 1 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} \end{matrix}$$

$P \leftarrow I$

↑
replace w, all 1

Inverse = ...

last row = $[0.1905 \quad 0.4286 \quad 0.3810]$

$\Phi_{\infty}(3)$

Starting at 2

avg # visits to 3 before returning to 2?

From 2

- back to 2, Prob. = $\frac{1}{3}$, 0 visits to 3
- go to 3, prob. $\frac{2}{3}$, ? visits to 3 before return to 2

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix} \end{matrix} \quad Q = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \quad M = \begin{matrix} & \begin{matrix} 1 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 3 \end{matrix} & \begin{bmatrix} 2 & 0 \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix} \end{matrix}$$

↑
to 2

So: answer is $0 \times \frac{1}{3} + \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$