Lec 1

Stochastic process : X_n, $n = 0, 1, 2, \cdots$

or $n = 1, 2, \cdots$

or X_t, $t \geq 0$

Will start w/ discrete time : $n = 0, 1, 2, \cdots$

X_n is valued in a state space $<$ finite or countably infinite

Eg. X_n iid or sums of iid

← Random walk

More general: correlated

Will study the case where $X_{n+1}, X_{n+2}, \cdots$ depend on $\cancel{X_n}$ only.

<u>Recall</u> $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$ $(P(A) > 0)$

A & B indep. means $P(B|A) = P(B)$

i.e. $P(A \cap B) = P(A) P(B)$

Lm. $P(A_0 \cap A_1 \cap \cdots \cap A_n)$

$= P(A_0)\,P(A_1 \mid A_0)\,P(A_2 \mid A_0 \cap A_1) \cdots P(A_n \mid A_0 \cap \cdots \cap A_{n-1})$

Pf. $n=1$: $P(A_0 \cap A_1) = P(A_0)\,P(A_1 \mid A_0)$ by def.

$n \to n+1$ induction (HW)

Application:

$P(X_0 = i_0, \ldots, X_n = i_n) = P(X_0 = i_0)\,P(X_1 = i_1 \mid X_0 = i_0)$

$\cdots P(X_n = i_n \mid X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1})$

So giving probabilities of "present | past" allow us to compute P(any event).

Markov: $P(X_n = i_n \mid X_0 = i_0, \ldots, X_{n-1} = i_{n-1}) = P(X_n = i_n \mid X_{n-1} = i_{n-1})$

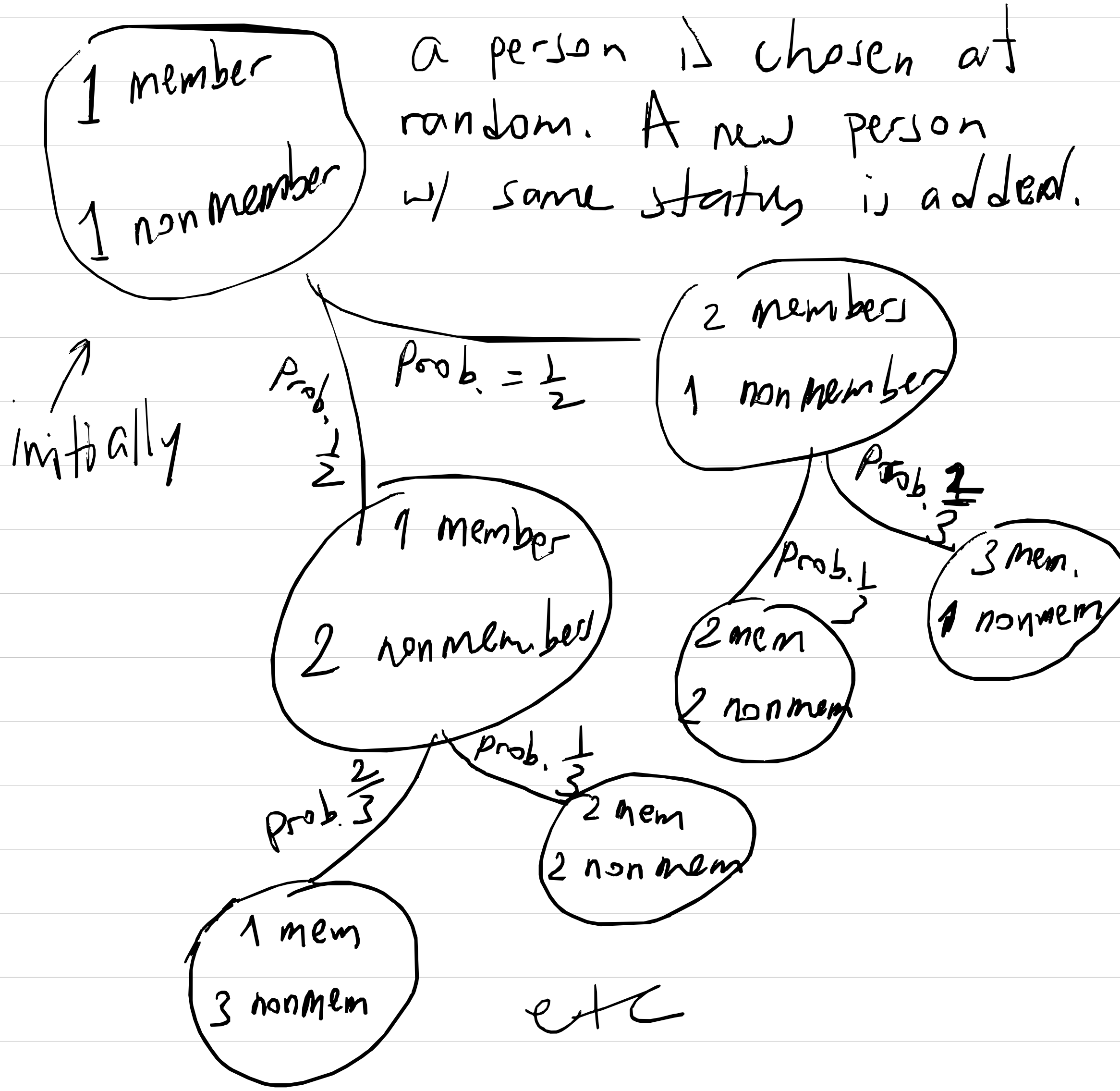
(P(future | present + past) = P(future | present))

Time homogenous Markov chain:

$P(X_n = j \mid X_{n-1} = i) = P_{ij}$ ← does not depend on n

Example (Polyá's urn)

a person is chosen at random. A new person w/ same status is added.



X_n = Status of person n : Not Markov

X_n = # members : Markov but not time homogenous

X_n = (# members, # non members) : Markov and time homogenous