

AI

How to generate a picture that has a "given style"?

Note

- Picture $\rightarrow$ matrix $k \times l$
- o/w if grey scale
- o/w RGB, encoded by $\{0, \dots, 255\}$ each
- So # possibilities is $256^{3 \times l}$ — huge!
- $k=l=256$: 65,536 pixels
 $\rightarrow$ 196,608 dimensions
- 512 $\rightarrow$ 786,432 dimensions
- 1024 $\rightarrow$ 3,145,728 dimensions
- 4k (2160 $\times$ 3840) : 25 million dimensions

Suppose Picasso paintings are all stored.
We want to generate a new random one
that looks like a Picasso.

Imagine Picasso is a random
generator of paintings.

So : probability mass function
we want to guess it. tough problem
Better : generate samples from it

Suppose Φ_0 is unknown.

But we have samples from it:
 $x_0^{(1)}, \dots, x_0^{(M)}$ (M samples)

We want to generate new samples.
But not by picking one of $x_0^{(i)}$ at random. (bootstrapping)

Suppose P is the transition matrix of some irreducible MC.

If we know: $\Phi_n = \Phi_0 P^n$

we can retrieve $\Phi_0 = \Phi_n P^{-n}$

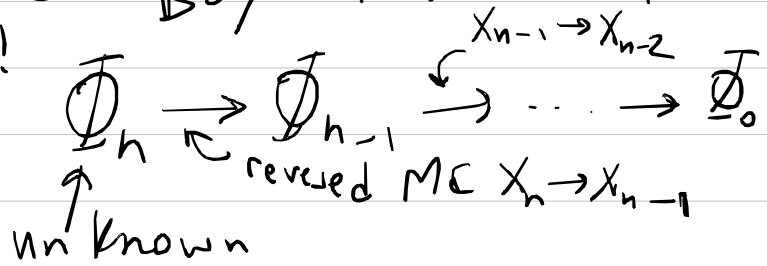
But to know Φ_n we need Φ_0 !

We know that starting at any $x_0^{(i)}$,
the distribution of X_n will
be $\rightarrow \Phi_\infty$ as $n \rightarrow \infty$.

So could try $\Phi_n \sim \Phi_\infty$

But $\Phi_\infty = \Phi_\infty P \Rightarrow \Phi_\infty P^{-1} = \Phi_\infty \Rightarrow \Phi_\infty P^{-n} = \Phi_\infty$
not Φ_0 $\nearrow$

Idea: use Bayes rule to reverse the MC!



but approximate w/ Φ_{∞} .

How to reverse the MC?

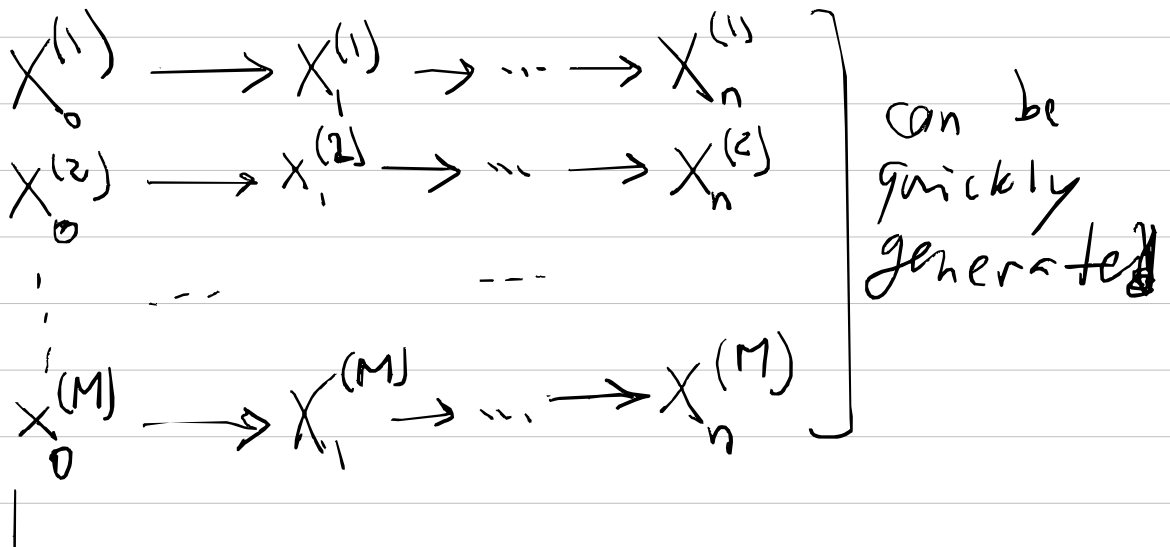
$$P(X_{n-1}=j | X_n=i) = \frac{P(X_n=i | X_{n-1}=j) P(X_{n-1}=j)}{P(X_n=i)}$$

Diagram illustrating the components of the equation:

- $P(X_{n-1}=j | X_n=i)$ is labeled "unknown".
- $P(X_n=i | X_{n-1}=j)$ is labeled "known".
- $P(X_{n-1}=j)$ is labeled "known".
- $P(X_n=i)$ is labeled "unknown".

call it $P_n(i, j)$

learn it from data.



Now, for each k :

use $(X_{k-1}^{(1)}, X_k^{(1)}) \dots (X_{k-1}^{(M)}, X_k^{(M)})$

to estimate $\bar{P}_k(i, j)$

Neural network:

$\bar{P}_k(i, j)$ unknown function of i & j

Want $\bar{P}_k(x_k^{(m)}, x_{k-1}^{(m)})$ as large as possible

w/ constraints: $\sum_j \bar{P}_k(i, j) = 1$

Linear regression:

$$\bar{P}_k(i, j) = \alpha_k i + \beta_k j + \gamma_k$$

estimate $\alpha_k, \beta_k, \gamma_k$

But $\bar{P}_k$ is not linear.

Neural Network: similar to $\bar{P}_k$ but
w/ a nonlinear form.

(still has parameters to estimate
and a [rather sophisticated]
method to estimate these parameters
from the data)

Once all these parameters are estimated, can use f_k as an approximation of p_k

Start MC at $\Phi_\infty \leftarrow$ known b/c we designed P
Run MC backward using f_k

to approximately sample from Φ_0 !

