Lec 13

$b(13; 0.6, 50)$

If $X \sim Bin(n, p)$, then the pmf of X is given by

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

NOTE: $\binom{n}{x}$ reads "n choose x", this tells us ________________________________

$$\binom{n}{x} = \frac{n(n-1)(n-2)\cdots(n-x+1)}{x!} = \frac{n!}{x!(n-x)!}$$

For example,

$$\binom{5}{2} = \frac{5 \times 4}{2!} = \frac{20}{2} = 10 \qquad \frac{5!}{2!\,3!} = \frac{120}{2 \times 6} = 10$$

Example 41. Each of six randomly selected cola drinkers is given a glass containing cola S and one containing cola F. Suppose there is no tendency among cola drinkers to prefer one cola to the other. Then $p = P(\text{a selected individual prefers } S) = 0.5$, so with $X =$ the number among the six who prefer S,

$$X \sim Bin(6, \tfrac{1}{2})$$

$$P(X=3) = b(3; 6, \tfrac{1}{2})$$

$$= \binom{6}{3}\left(\tfrac{1}{2}\right)^3 \left(1-\tfrac{1}{2}\right)^{6-3}$$

$$= \frac{6 \times 5 \times 4}{3!} \cdot \left(\tfrac{1}{2}\right)^6 = \ldots$$

$$P(X \le 1) = P(X=0) + P(X=1) = \binom{6}{0}\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^6 + \binom{6}{1}\left(\tfrac{1}{2}\right)^1\left(\tfrac{1}{2}\right)^5$$

$$= \left(\tfrac{1}{2}\right)^6 + 6\left(\tfrac{1}{2}\right)^6 = 7\left(\tfrac{1}{2}\right)^6$$

Using Binomial tables

Even for a relatively small value of n, the computation of binomial probabilities can be tedious. Appendix Table A.1 tabulates the cdf ______________________ for $n = 5, 10, 15, 20, 25$ in combination with selected values of p corresponding to different columns of the table.

> **Notation:** For $X \sim Bin(n, p)$, the cdf will be denoted by
>
> $B(x; n, p) = P(X \leq x)$

Example 42. Suppose that 20% of all copies of a textbook fail a certain binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. Then

X has $Bin(15, 0.2)$ distribution.

1. The probability that at most 8 fail the test is

$P(X \leq 8) = 0.999$ (from table)

2. The probability that exactly 8 fail is

$$P(X = 8) = P(X \leq 8) - P(X \leq 7)$$
$$= 0.999 - 0.996$$

3. The probability that at least 8 fail is

$$P(X \geq 8) = 1 - P(X \leq 7)$$
$$= 1 - 0.996$$

4. The probability that between 4 and 7, inclusive, fail is

$$P(4 \le X \le 7) = P(X \le 7) - P(X \le 3)$$
$$= 0.996 - 0.648$$

A-2 Appendix Tables

$P(X \le 6) = 0.982$

$P(X = 6) = P(X \le 6) - P(X \le 5)$

$= 0.982 - 0.939$

Table A.1 Cumulative Binomial Probabilities

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

c. $n = 15$ $\quad$ $p = 0.2$

		p														
		0.01	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**	**0.99**
	0	.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
	2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
	4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
	5	1.000	1.000	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000	.000
	6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
x	7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
	8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
	9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
	10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
	11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000

$P(X \ge 6) = 1 - P(X < 6)$

$= 1 - P(X \le 5) = 1 - 0.939$

$P(3 \le X \le 6) = P(X \le 6) - P(X \le 2)$

$= 0.982 - 0.398$

$P(3 < X \le 6) = P(X \le 6) - P(X \le 3)$

$n = 15,\ p = 0.2$ $\quad$ $= 0.982 - 0.648$

$= 15 \times (0.2)^{14} \times 0.8 + 1 \times (0.2)^{15}$

$P(X \le 13) = 1 - P(X \ge 14) = b(14; 15, 0.2) + b(15; 15, 0.2)$